X Revision – TRIGONOMETRY

1) Express the trigonometric ratios Sin A, Sec A and tan A in terms of Cot A.

2) Write all the other trigonometric ratios of ∠A in terms of Sec A

3) If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, $0<\theta<90°$, find the values of $\cos\theta$ and $\tan\theta$.

4) If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

5) Evaluate : (i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cdot \cos 65° + \cos 25° \cdot \sin 65°$

6) If $A+B = 90°$, Prove that $\sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$

7) Prove that : (i) $\tan 1° \cdot \tan 2° \cdot \tan 3° \ldots \tan 89° = 1$

(ii) $\cos 1° \cdot \cos 2° \cdot \cos 3° \ldots \cos 180° = 0$

8) $9\sec^2 A - 9\tan^2 A =$ ____ (a) 1 (b) 9 (c) 8 (d) 0

9) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta) =$ ____ (a) 0 (b) 1 (c) 2 (d) −1

10) $(\sec A+\tan A)(1-\sin A) =$ ____ (a) Sec A (b) Sin A (c) Cosec A (d) Cos A

11) $\frac{1+\tan^2 A}{1+\cot^2 A} =$ ____ (a) $\sec^2 A$ (b) −1 (c) $\cot^2 A$ (d) $\tan^2 A$

12) Prove that (i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $(\sec A - \tan A)^2 = \frac{1-\sin A}{1+\sin A}$

(iii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iv) $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

(v) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta \operatorname{cosec}\theta$

(vi) $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

(vii) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(viii) $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$

(ix) $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$

(x) $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1+\cos A}{\sin A}$

(xi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(xii) $\sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}} = 2\operatorname{cosec} A$

(xiii) $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta$

(xiv) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(xv) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(xvi) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(xvii) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

(xviii) $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$

(xix) $\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$

(xx) If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that

$$\cos\theta = \frac{p^2-1}{p^2+1}$$

X Revision – TRIGONOMETRY answers

1) $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1+\cot^2 A}}$

$\sec A = \sqrt{1+\tan^2 A} = \sqrt{1+\frac{1}{\cot^2 A}} = \sqrt{\frac{\cot^2 A+1}{\cot^2 A}} = \frac{\sqrt{\cot^2 A+1}}{\cot A}$

$\tan A = \frac{1}{\cot A}$

2) $\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{1}{\sec^2 A}} = \sqrt{\frac{\sec^2 A-1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A-1}}{\sec A}$

$\cos A = \frac{1}{\sec A}$

$\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{\sec^2 A-1}}{\sec A} \div \frac{1}{\sec A} = \frac{\sqrt{\sec^2 A-1}}{\cancel{\sec A}} \times \cancel{\sec A}$

$= \sqrt{\sec^2 A-1}$

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A-1}}$

$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A-1}}$

3) $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{a^2}{a^2+b^2}} = \sqrt{\frac{\cancel{a^2}+b^2-\cancel{a^2}}{a^2+b^2}} = \sqrt{\frac{b^2}{a^2+b^2}}$

$= \frac{b}{\sqrt{a^2+b^2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a}{\cancel{\sqrt{a^2+b^2}}} \times \frac{\cancel{\sqrt{a^2+b^2}}}{b} = \frac{a}{b}$

4) Given, $\cot\theta = \frac{15}{8}$

$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = \frac{\cancel{2}(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)\times\cancel{2}(1-\cos\theta)}$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$$

[$\because \cos^2\theta = 1-\sin^2\theta$; $\sin^2\theta = 1-\cos^2\theta$; $\cot\theta = \cos\theta/\sin\theta$]

5)

(i) $$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$= \frac{\sin^2 63^\circ + \cos^2(90^\circ - 27^\circ)}{\sin^2(90^\circ - 17^\circ) + \cos^2 73^\circ}$$ [$\cos(90^\circ-\theta) = \sin\theta$; $\sin(90^\circ-\theta) = \cos\theta$]

$$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} = \frac{1}{1} = 1$$ [$\sin^2\theta + \cos^2\theta = 1$]

(ii) $\sin 25^\circ . \cos 65^\circ + \cos 25^\circ . \sin 65^\circ$

$= \sin 25^\circ . \sin(90^\circ - 65^\circ) + \cos 25^\circ . \cos(90^\circ - 65^\circ)$

$= \sin 25^\circ . \sin 25^\circ + \cos 25^\circ . \cos 25^\circ$ [$\sin(90^\circ-\theta) = \cos\theta$; $\cos(90^\circ-\theta) = \sin\theta$]

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]

6) Given, $A+B = 90^\circ$

$\Rightarrow B = 90^\circ - A$

Thus, $$\sqrt{\frac{\tan A . \tan(90^\circ - A) + \tan A . \cot(90^\circ - A)}{\sin A . \sec(90^\circ - A)} - \frac{\sin^2(90^\circ - A)}{\cos^2 A}}$$

$$= \sqrt{\frac{\tan A . \cot A + \tan A . \tan A}{\sin A . \operatorname{cosec} A} - \frac{\cos^2 A}{\cos^2 A}}$$ [$\tan(90^\circ-\theta) = \cot\theta$; $\cot(90^\circ-\theta) = \tan\theta$; $\sin(90^\circ-\theta) = \cos\theta$; $\sec(90^\circ-\theta) = \operatorname{cosec}\theta$]

$$= \sqrt{\frac{1 + \tan^2 A}{1} - 1}$$

$$= \sqrt{1 + \tan^2 A - 1}$$ [$\because \tan\theta . \cot\theta = 1$; $\sin\theta . \operatorname{cosec}\theta = 1$]

$$= \sqrt{\tan^2 A}$$

$$= \tan A$$

7) (i) LHS, $\tan 1^\circ . \tan 2^\circ . \tan 3^\circ \ldots \tan 89^\circ$

$= (\tan 1^\circ . \tan 89^\circ).(\tan 2^\circ . \tan 88^\circ) \ldots \tan 45^\circ$

$= (\tan 1^\circ . \cot(90^\circ - 89^\circ))(\tan 2^\circ . \cot(90^\circ - 88^\circ)) \ldots \tan 45^\circ$

$[\because \cot(90^\circ - \theta) = \tan\theta]$

$= (\tan 1^\circ . \cot 1^\circ)(\tan 2^\circ . \cot 2^\circ) \ldots \tan 45^\circ$

$= 1 \times 1 \times 1 \ldots \times 1$ $[\because \tan\theta . \cot\theta = 1 ; \tan 45^\circ = 1]$

$= \underline{\underline{1}}$, RHS

8) $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \underline{\underline{9}}$ (b)

9) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$ $[\because \tan\theta = \frac{\sin\theta}{\cos\theta} ; \sec\theta = \frac{1}{\cos\theta} ;$

$= \left(\frac{(\cos\theta + \sin\theta) + 1}{\cos\theta}\right)\left(\frac{(\sin\theta + \cos\theta) - 1}{\sin\theta}\right)$ $\cot\theta = \frac{\cos\theta}{\sin\theta} ; \operatorname{cosec}\theta = \frac{1}{\sin\theta}]$

$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta . \cos\theta}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta . \cos\theta - 1}{\sin\theta . \cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{\cancel{1} + 2\sin\theta\cos\theta - \cancel{1}}{\sin\theta . \cos\theta} = \frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} = \underline{\underline{2}}$ (c)

10) $(\sec A + \tan A)(1 - \sin A)$

$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$ $[\because \sec\theta = 1/\cos\theta ; \tan\theta = \sin\theta/\cos\theta]$

$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$

$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$

$= \frac{1 - \sin^2 A}{\cos A}$

$= \frac{\cos^{\cancel{2}} A}{\cancel{\cos A}}$ $[\because \cos^2\theta = 1 - \sin^2\theta]$

$= \cos A$ (d)

11) $\dfrac{1+\tan^2 A}{1+\cot^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A} = \dfrac{1}{\cos^2 A} \times \sin^2 A = \tan^2 A$ (d)

12) (i) LHS, $(\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \quad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta};\ \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)\cancel{(1-\cos\theta)}}{(1+\cos\theta)\cancel{(1-\cos\theta)}} = \frac{1-\cos\theta}{1+\cos\theta}, \text{RHS}$$

$[\sin^2\theta = 1-\cos^2\theta]$

(ii) LHS, $(\sec A - \tan A)^2$

$$= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 \quad \left[\because \sec A = \frac{1}{\cos A};\ \tan A = \frac{\sin A}{\cos A}\right]$$

$$= \left(\frac{1-\sin A}{\cos A}\right)^2$$

$$= \frac{(1-\sin A)^2}{\cos^2 A}$$

$$= \frac{(1-\sin A)^2}{1-\sin^2 A} \quad [\because \cos^2\theta = 1-\sin^2\theta]$$

$$= \frac{\cancel{(1-\sin A)}(1-\sin A)}{\cancel{(1-\sin A)}(1+\sin A)} = \frac{1-\sin A}{1+\sin A}, \text{RHS}$$

(iii) LHS, $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{(\sin^2 A + \cos^2 A) + 1 + 2\sin A}{\cos A(1+\sin A)} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)} = \frac{2+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2\cancel{(1+\sin A)}}{\cos A\cancel{(1+\sin A)}} = \frac{2}{\cos A} = 2\sec A, \text{RHS} \quad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

(iv) LHS, $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A}$

$= \dfrac{\cos A}{1-\dfrac{\sin A}{\cos A}} + \dfrac{\sin A}{1-\dfrac{\cos A}{\sin A}}$ $\left[\because \tan A = \dfrac{\sin A}{\cos A}; \cot A = \dfrac{\cos A}{\sin A}\right]$

$= \dfrac{\cos^2 A}{\cos A - \sin A} + \dfrac{\sin^2 A}{\sin A - \cos A}$

$= \dfrac{\cos^2 A}{\cos A - \sin A} - \dfrac{\sin^2 A}{\cos A - \sin A}$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos A - \sin A} = \dfrac{(\cos A + \sin A)\cancel{(\cos A - \sin A)}}{\cancel{\cos A - \sin A}}$

$= \sin A + \cos A$, RHS

(v) LHS, $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$= \dfrac{\tan\theta}{1-\dfrac{1}{\tan\theta}} + \dfrac{1}{\tan\theta(1-\tan\theta)}$ $[\because \cot\theta = 1/\tan\theta]$

$= \dfrac{\tan^2\theta}{\tan\theta - 1} + \dfrac{1}{\tan(1-\tan\theta)}$

$= \dfrac{\tan^2\theta}{\tan\theta - 1} - \dfrac{1}{\tan\theta(\tan\theta - 1)}$ $[a^3 - b^3 = (a-b)(a^2+ab+b^2)]$

$= \dfrac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)} = \dfrac{\cancel{(\tan\theta - 1)}(\tan^2\theta + \tan\theta + 1)}{\tan\theta\,\cancel{(\tan\theta - 1)}}$

$= \dfrac{\tan^2\theta + \tan\theta + 1}{\tan\theta} = \dfrac{\tan^2\theta}{\tan\theta} + \dfrac{\tan\theta}{\tan\theta} + \dfrac{1}{\tan\theta}$

$= \tan\theta + 1 + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + 1 + \dfrac{\cos\theta}{\sin\theta}$ $[\because \tan\theta = \sin\theta/\cos\theta; \cot\theta = \cos\theta/\sin\theta]$

$= 1 + \left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right) = 1 + \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= 1 + \dfrac{1}{\sin\theta\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 + \sec\theta . \operatorname{cosec}\theta$ // $\left[\because \dfrac{1}{\cos\theta} = \sec\theta; \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$

$=$ RHS

LHS,

(vi) $\dfrac{\operatorname{Cosec} A}{\operatorname{Cosec} A - 1} + \dfrac{\operatorname{Cosec} A}{\operatorname{Cosec} A + 1}$

$= \operatorname{Cosec} A \left[\dfrac{1}{\operatorname{Cosec} A - 1} + \dfrac{1}{\operatorname{Cosec} A + 1} \right]$

$= \operatorname{Cosec} A \left[\dfrac{\operatorname{Cosec} A + \cancel{1} + \operatorname{Cosec} A - \cancel{1}}{(\operatorname{Cosec} A - 1)(\operatorname{Cosec} A + 1)} \right]$

$= \operatorname{Cosec} A \times \dfrac{2 \operatorname{Cosec} A}{\operatorname{Cosec}^2 A - 1}$ $[(a+b)(a-b) = a^2 - b^2]$

$= \dfrac{2 \operatorname{Cosec}^2 A}{\operatorname{Cosec}^2 A - 1} = \dfrac{2 \operatorname{Cosec}^2 A}{\operatorname{Cot}^2 A}$ $[\because \operatorname{Cot}^2\theta = \operatorname{Cosec}^2\theta - 1]$

$= 2 \dfrac{1}{\operatorname{Sin}^2 A} \times \dfrac{\operatorname{Sin}^2 A}{\operatorname{Cos}^2 A}$ $\left[\because \operatorname{Cosec}\theta = \dfrac{1}{\sin\theta} ; \operatorname{Cot}\theta = \dfrac{\cos\theta}{\sin\theta}\right]$

$= \dfrac{2}{\operatorname{Cos}^2 A} = 2 \operatorname{Sec}^2 A$, RHS $\left[\because \operatorname{Sec}\theta = \dfrac{1}{\cos\theta}\right]$

(vii) LHS, $\dfrac{1 + \operatorname{Sec} A}{\operatorname{Sec} A} = \dfrac{1 + \dfrac{1}{\operatorname{Cos} A}}{\dfrac{1}{\operatorname{Cos} A}}$ $[\because \operatorname{Sec}\theta = 1/\cos\theta]$

$= \dfrac{\operatorname{Cos} A + 1}{\cancel{\operatorname{Cos} A}_{\,1}} \times \dfrac{\cancel{\operatorname{Cos} A}^{\,1}}{1}$

$= 1 + \operatorname{Cos} A$

$= \dfrac{(1 + \operatorname{Cos} A)(1 - \operatorname{Cos} A)}{(1 - \operatorname{Cos} A)}$ [X numerator and denominator by $(1-\operatorname{Cos} A)$]

$= \dfrac{1 - \operatorname{Cos}^2 A}{1 - \operatorname{Cos} A} = \dfrac{\operatorname{Sin}^2 A}{1 - \operatorname{Cos} A}$, RHS $[\because \sin^2\theta = 1 - \cos^2\theta]$

(viii) LHS, $\dfrac{1 + \operatorname{Cos} A}{\operatorname{Sin} A}$

$= \dfrac{(1 + \operatorname{Cos} A)(1 - \operatorname{Cos} A)}{\operatorname{Sin} A\,(1 - \operatorname{Cos} A)}$ [X numerator and denominator by $(1-\operatorname{Cos} A)$]

$= \dfrac{1 - \operatorname{Cos}^2 A}{\operatorname{Sin} A\,(1 - \operatorname{Cos} A)}$

$= \dfrac{\operatorname{Sin}^{\cancel{2}\,1} A}{\cancel{\operatorname{Sin} A}\,(1 - \operatorname{Cos} A)} = \dfrac{\operatorname{Sin} A}{1 - \operatorname{Cos} A}$, RHS $[\sin^2\theta = 1 - \cos^2\theta]$

(ix) LHS, $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \dfrac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\sec\theta + \tan\theta)\,[\cancel{1 - \sec\theta + \tan\theta}]}{\cancel{\tan\theta - \sec\theta + 1}}$

$= \sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ $[\because \sec\theta = 1/\cos\theta;\ \tan\theta = \sin\theta/\cos\theta]$

$= \dfrac{1 + \sin\theta}{\cos\theta}$, RHS //

(x) LHS, $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$ $[\because \operatorname{cosec}^2 A - \cot^2 A = 1]$

$= \dfrac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{(\operatorname{cosec} A + \cot A)\,[\cancel{1 - \operatorname{cosec} A + \cot A}]}{\cancel{\cot A - \operatorname{cosec} A + 1}}$

$= \operatorname{cosec} A + \cot A$ $\left[\because \operatorname{cosec} A = 1/\sin A;\ \cot A = \dfrac{\cos A}{\sin A}\right]$

$= \dfrac{1}{\sin A} + \dfrac{\cos A}{\sin A} = \dfrac{1 + \cos A}{\sin A}$, RHS //

(xi) LHS, $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sqrt{\dfrac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$ [× numerator and denominator by $(1 + \sin A)$]

$= \sqrt{\dfrac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\dfrac{(1 + \sin A)^2}{\cos^2 A}}$ $[\because \cos^2\theta = 1 - \sin^2\theta]$

$= \dfrac{1 + \sin A}{\cos A}$

$= \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A$, RHS $\left[\because \sec\theta = \dfrac{1}{\cos\theta};\ \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

LHS,

(xii) $\sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}}$

$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$

$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} + \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$

$[\because \sin^2\theta = 1-\cos^2\theta]$

$= \frac{1-\cos A}{\sin A} + \frac{1+\cos A}{\sin A}$

$= \frac{1-\cancel{\cos A}+1+\cancel{\cos A}}{\sin A} = \frac{2}{\sin A} = 2\operatorname{cosec} A$, RHS

$[\because \operatorname{cosec}\theta = 1/\sin\theta]$

(xiii) LHS, $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$= \sqrt{\frac{(\sec\theta-1)(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}} + \sqrt{\frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$

$= \sqrt{\frac{(\sec\theta-1)^2}{\sec^2\theta-1}} + \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}} = \sqrt{\frac{(\sec\theta-1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$

$[\because \tan^2\theta = \sec^2\theta - 1]$

$= \frac{\sec\theta-1}{\tan\theta} + \frac{\sec\theta+1}{\tan\theta}$

$= \frac{\sec\theta-\cancel{1}+\sec\theta+\cancel{1}}{\tan\theta} = \frac{2\sec\theta}{\tan\theta} = 2\times\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}$

$= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$, RHS $\quad [\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}] \quad [\sec\theta = \frac{1}{\cos\theta};\ \tan\theta = \frac{\sin\theta}{\cos\theta}]$

(xiv) LHS, $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

$$= \frac{\sin\theta\,(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\cancel{(\cos^2\theta - \sin^2\theta)}^{1}}{\cancel{(\cos^2\theta - \sin^2\theta)}_{1}}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta, \text{RHS}$$ $[\because \tan\theta = \sin\theta/\cos\theta]$

(xv) LHS, $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A . \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A . \sec A$

$= (\sin^2 A + \cos^2 A) + 2\cancel{\sin A} \times \frac{1}{\cancel{\sin A}_1} + 2\cancel{\cos A} \times \frac{1}{\cancel{\cos A}_1} + \operatorname{cosec}^2 A + \sec^2 A$

$= 1 + 2 + 2 + 1 + \cot^2 A + 1 + \tan^2 A = 7 + \cot^2 A + \tan^2 A$, RHS

$[\because \sin^2\theta + \cos^2\theta = 1; \operatorname{cosec}\theta = 1/\sin\theta; \sec A = 1/\cos A; \operatorname{cosec}^2 A = 1 + \cot^2 A;$
$1 + \tan^2 A = \sec^2 A]$

(xvi) LHS, $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$ $\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}\right]$

$$= \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A}$$

$$= \frac{\cancel{\cos^2 A}}{\cancel{\sin A}_1} \times \frac{\cancel{\sin^2 A}}{\cancel{\cos A}_1} = \sin A \cos A$$ $[\because \cos^2\theta = 1 - \sin^2\theta$
$\sin^2\theta = 1 - \cos^2\theta]$

$$= \frac{\sin A . \cos A}{1} = \frac{\sin A . \cos A}{\sin^2 A + \cos^2 A} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A . \cos A}}$$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{1}{\frac{\cancel{\sin^2 A}}{\cancel{\sin A}\cos A} + \frac{\cancel{\cos^2 A}}{\sin A\,\cancel{\cos A}}} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A}, \text{RHS}$$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}; \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

(xvii) LHS, $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{1}{\cos^2 A} \times \sin^2 A = \tan^2 A$

$[\because \sec^2\theta = 1 + \tan^2\theta; \operatorname{cosec}^2\theta = 1 + \cot^2\theta;$
$\tan\theta = \sin\theta/\cos\theta]$

Also, $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$ $\left[\because \cot\theta = \frac{1}{\tan\theta}\right]$

$= \left(\frac{\tan A(1-\tan A)}{\tan A - 1}\right)^2 = \left(\frac{-\tan A\cancel{(1-\tan A)}}{\cancel{1-\tan A}}\right)^2$

$= (-\tan A)^2 = \underline{\underline{\tan^2 A}}$

(xviii) LHS, $\frac{1}{\sec A + \tan A} - \frac{1}{\cos A}$

$= \frac{\sec^2 A - \tan^2 A}{\sec A + \tan A} - \sec A$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \frac{\cancel{(\sec A + \tan A)}(\sec A - \tan A)}{\cancel{\sec A + \tan A}} - \sec A$

$= (\sec A - \tan A) - \sec A$

$= \sec A - (\tan A + \sec A)$

$= \sec A - \frac{(\sec A + \tan A)}{\sec^2 A - \tan^2 A}$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \sec A - \frac{\cancel{\sec A + \tan A}}{(\sec A - \tan A)\cancel{(\sec A + \tan A)}}$

$= \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$, RHS //

(xix) LHS, $\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A}$

$= \frac{\operatorname{cosec}^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A} - \operatorname{cosec} A$ $[\operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$= \frac{(\operatorname{cosec} A + \cot A)\cancel{(\operatorname{cosec} A - \cot A)}}{\cancel{\operatorname{cosec} A - \cot A}} - \operatorname{cosec} A$

$= \operatorname{cosec} A + (\cot A - \operatorname{cosec} A)$

$= \operatorname{cosec} A - (\operatorname{cosec} A - \cot A)$

$= \operatorname{cosec} A - \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec}^2 A - \cot^2 A}$ $[\operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$= \operatorname{cosec} A - \frac{\cancel{\operatorname{cosec} A - \cot A}}{(\operatorname{cosec} A + \cot A)\cancel{(\operatorname{cosec} A - \cot A)}}$

$$= \operatorname{Cosec} A - \frac{1}{\operatorname{Cosec}A + \operatorname{Cot}A}$$

$$= \frac{1}{\sin A} - \frac{1}{\operatorname{Cosec}A + \operatorname{Cot}A}, \text{ RHS} \quad [\because \operatorname{Cosec}\theta = 1/\sin\theta]$$

(XX) Given, $\operatorname{Cosec}\theta + \operatorname{Cot}\theta = p \longrightarrow (1)$

$$\Rightarrow \frac{1}{\operatorname{Cosec}\theta + \operatorname{Cot}\theta} = \frac{1}{p}$$

$$\Rightarrow \frac{\operatorname{Cosec}^2\theta - \operatorname{Cot}^2\theta}{\operatorname{Cosec}\theta + \operatorname{Cot}\theta} = \frac{1}{p} \quad [\because \operatorname{Cosec}^2\theta - \operatorname{Cot}^2\theta = 1]$$

$$\Rightarrow \frac{\cancel{(\operatorname{Cosec}\theta + \operatorname{Cot}\theta)}(\operatorname{Cosec}\theta - \operatorname{Cot}\theta)}{\cancel{\operatorname{Cosec}\theta + \operatorname{Cot}\theta}} = \frac{1}{p}$$

$$\Rightarrow \operatorname{Cosec}\theta - \operatorname{Cot}\theta = \frac{1}{p} \longrightarrow (2)$$

eq:(1)+eq:(2), $2\operatorname{Cosec}\theta = p + \frac{1}{p}$

$$\Rightarrow 2\operatorname{Cosec}\theta = \frac{p^2+1}{p}$$

$$\Rightarrow \operatorname{Cosec}\theta = \frac{p^2+1}{2p} \longrightarrow (3)$$

eq:(1)−(2), $2\operatorname{Cot}\theta = p - \frac{1}{p}$

$$\Rightarrow 2\operatorname{Cot}\theta = \frac{p^2-1}{p}$$

$$\Rightarrow \operatorname{Cot}\theta = \frac{p^2-1}{2p} \longrightarrow (4)$$

eq: $\frac{(4)}{(3)}$, $\frac{\operatorname{Cot}\theta}{\operatorname{Cosec}\theta} = \frac{p^2-1}{\cancel{2p}} \times \frac{\cancel{2p}}{p^2+1} = \frac{p^2-1}{p^2+1}$

$$\frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta} = \frac{p^2-1}{p^2+1} \quad [\because \operatorname{Cot}\theta = \cos\theta/\sin\theta; \ \operatorname{Cosec}\theta = 1/\sin\theta]$$

$$\therefore \cos\theta = \frac{p^2-1}{p^2+1}$$