IX Revision – (Polynomials)

MCQs

1) Zero of the polynomial $p(x)$, where $p(x) = ax+1, a \neq 0$ is (a) 1 (b) $-a$ (c) 0 (d) $-\frac{1}{a}$

2) If polynomial $p(x) = 3x^4 - 4x^3 - 3x - 1$ is divided by $(x-1)$, then remainder is (a) 3 (b) -4 (c) -1 (d) $p(1)$

3) To evaluate $(99)^3$, best option is expansion of
(a) $(95+4)^3$ (b) $(90+9)^3$ (c) $(100-1)^3$ (d) $(98+1)^3$

4) Are -3 and 3 zeroes of the polynomial $x+3$?
(a) only -3 (b) only 3 (c) none (d) both

5) $(x+1)$ is a factor of the polynomial
(a) x^3+x^2-x+1 (b) x^3+x^2+x+1 (c) $x^4+x^3+x^2+1$
(d) $x^4+3x^3+3x^2+x+1$

6) If $64x^2 - y = \left(8x+\frac{1}{2}\right)\left(8x-\frac{1}{2}\right)$, then the value of y is
(a) 0 (b) $1/\sqrt{2}$ (c) $1/4$ (d) $1/2$

7) The coefficient of x in the expansion of $(x+3)^3$ is
(a) 1 (b) 9 (c) 18 (d) 27.

8) Volume of a cuboid is $3x^2 - 27$. Then possible dimensions are (a) 3, 3, 3 (b) $3(x-3)(x+3)$ (c) $3, x^2, 27x$
(d) $3, x^2, -27x$

9) Without multiplying directly, find the product of 103×107
(a) 11021 (b) 12051 (c) 12091 (d) 10918

10) If $(x-1)$ is a factor of $kx^2 - \sqrt{2}x + 1$, then value of k is
(a) 1 (b) $\sqrt{2}+1$ (c) $\sqrt{2}-1$ (d) $-\sqrt{2}-1$

11) $(x-2y)^3 + (2y-3z)^3 + (3z-x)^3$ is equal to
(a) $(x-2y)(2y-3z)(3z-x)$
(b) $2(x-2y)(2y-3z)(3z-x)$
(c) $3(x-2y)(2y-3z)(3z-x)$
(d) $3(x-2y)(3z-x)$

12) To factorize $x^3+13x^2+32x+20$, we
(a) split the middle term (b) Combine x^3+13x^2 and $32x+20$
(c) Combine x^3+32x and $13x^2+20$
(d) use factor theorem to find factors.

13) Using factor theorem, factorise the polynomial $x^4 - x^3 - 7x^2 + x + 6$.

14) What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that result is exactly divisible by $x^2 + x - 2$?

15) If $x^2 + \frac{1}{x^2} = 66$, find the value of $x^3 - \frac{1}{x^3}$

16) If $x^2 - 1$ is a factor of $px^4 + qx^3 + rx^2 + Sx + u$, show that $p + r + u = q + S = 0$

17) When $f(y) = y^4 - 4y^3 + 8y^2 - my + n$ is divided by $y+1$ and $y-1$, we get remainder as 10 and 16 respectively. Find the remainder if $f(y)$ is divided by $(y-3)$

18) Simplify : $(p+q)^3 - (p-q)^3 - 6q(p^2 - q^2)$

19) If $3x - 2y = 13$ and $xy = 5$, find the value of $27x^3 - 8y^3$

20) Without actually calculating the cubes, find the value of

(i) $(-14)^3 + 8^3 + 6^3$

(ii) $27^3 + (-14)^3 + (-13)^3$

IX Revision (POLYNOMIALS - Answers)

1) Put $p(x) = 0$

$\Rightarrow ax + 1 = 0$

$\Rightarrow x = -\frac{1}{a}$ (d)

2) $p(1)$ (d)

3) $(100-1)^3$ (c)

4) put $x+3=0$

$\Rightarrow x = -3$

only -3 (a)

5) put $(x+1) = 0$

$\therefore x = -1$

(a) $p(x) = x^3 + x^2 - x + 1$

$p(-1) = -1+1+1+1 = 2 \neq 0$

(b) $p(x) = x^3 + x^2 + x + 1$

$p(-1) = -1+1-1+1 = 0$

(c) $p(x) = x^4 + x^3 + x^2 + 1$

$p(-1) = 1-1+1+1 = 2 \neq 0$

(d) $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = 1-3+3-1+1 = 1 \neq 0$

Thus $(x+1)$ is a factor of x^3+x^2+x+1 (b)

6) $(a+b)(a-b) = a^2 - b^2$

$\left(8x + \frac{1}{2}\right)\left(8x - \frac{1}{2}\right) = (8x)^2 - \left(\frac{1}{2}\right)^2 = 64x^2 - \frac{1}{4}$

$\therefore y = \frac{1}{4}$ (c)

7) $(x+3)^3 = x^3 + 3\times x^2 \times 3 + 3 \times x \times 3^2 + 3^3$

$[(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= x^3 + 9x^2 + 27x + 27$

Thus coefficient of x is 27 (d)

8) $3x^2 - 27 = 3(x^2 - 9) = 3(x+3)(x-3)$ (b)

$[a^2 - b^2 = (a+b)(a-b)]$

9) $103 \times 107 = (100+3)(100+7)$

$[(x+a)(x+b) = x^2 + (a+b)x + ab]$; $x = 100, a = 3, b = 7$

$= 100^2 + (3+7)\times 100 + 3\times 7$

$= 10000 + 1000 + 21$

$= 11021$ (a)

$$\begin{array}{r} 10000 \\ 1000 \\ 21 \\ \hline 11021 \end{array}$$

10) Let $p(x) = kx^2 - \sqrt{2}x + 1$

Since $(x-1)$ is a factor of $p(x)$,

then $p(1) = 0$

$\Rightarrow k(1)^2 - \sqrt{2} \times 1 + 1 = 0$

$\Rightarrow k - \sqrt{2} + 1 = 0$

$\therefore k = \sqrt{2} - 1$ (c)

11) [If $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$]

Checking:- $\cancel{x} - \cancel{2y} + \cancel{2y} - \cancel{3z} + \cancel{3z} - \cancel{x} = 0$

$\therefore (x-2y)^3 + (2y-3z)^3 + (3z-x)^3 = 3(x-2y)(2y-3z)(3z-x)$ (c)

12) Use factor theorem to find factors (d)

13) Let $p(x) = x^4 - x^3 - 7x^2 + x + 6$

Factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

$p(1) = 1^4 - 1^3 - 7(1)^2 + (1) + 6$

$= 1 - 1 - 7 + 1 + 6 = 8 - 8 = 0$

Thus $(x-1)$ is a factor of $p(x)$

$p(-1) = (-1)^4 - (-1)^3 - 7(-1)^2 + (-1) + 6$

$= 1 + 1 - 7 - 1 + 6 = 8 - 8 = 0$

Thus $(x+1)$ is a factor of $p(x)$

Then $(x+1)(x-1) = x^2 - 1$ is also a factor of $p(x)$.

On dividing $p(x)$ by (x^2-1),

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x^2 - 1 & x^4 - x^3 - 7x^2 + x + 6 \\ & \overset{(-)}{\cancel{x^4}} + 0x^3 \overset{(+)}{-} x^2 \\ \hline & \cancel{-x^3} - 6x^2 + \cancel{x} + 6 \\ & \overset{(+)}{-}\cancel{x^3} + 0x^2 \overset{(-)}{+} \cancel{x} \\ \hline & -\cancel{6x^2} + \cancel{6} \\ & \overset{(+)}{-}\cancel{6x^2} \overset{(-)}{+} \cancel{6} \\ \hline & 0 \end{array}$$

quotient $= x^2 - x - 6$

remainder $= 0$

Using division algorithm,

$p(x) = (x^2-1)(x^2-x-6)$

$= (x^2-1)(x-3)(x+2)$

$\therefore p(x) = (x+1)(x-1)(x-3)(x+2)$

S: -1 P: -6

$-3, 2$

14) Let $p(x) = x^4 + 2x^3 - 2x^2 + x - 1$.
On dividing $p(x)$ by $(x^2 + x - 2)$,

$$\begin{array}{r}
x^2 + x - 1 \\
x^2 + x - 2 \overline{\big) x^4 + 2x^3 - 2x^2 + x - 1} \\
\overset{(-)}{} \cancel{x^4} \overset{(-)}{+} x^3 \overset{(+)}{-} 2x^2 \\
\hline
x^3 + 0x^2 + x - 1 \\
\overset{(-)}{} \cancel{x^3} \overset{(-)}{+} x^2 \overset{(+)}{-} 2x \\
\hline
-x^2 + 3x - 1 \\
\overset{(+)}{-} \cancel{x^2} \overset{(+)}{-} x \overset{(-)}{+} 2 \\
\hline
4x - 3
\end{array}$$

$\therefore$ The required polynomial to be added is $-4x + 3$ //

15) $[(a-b)^2 = a^2 + b^2 - 2ab]$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$
$$= 66 - 2$$
$$= 64$$

$\therefore x - \frac{1}{x} = \sqrt{64} = 8 \longrightarrow (1)$

$[a^3 - b^3 = (a-b)^3 + 3ab(a-b)]$

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3 \times \cancel{x} \times \frac{1}{\cancel{x}}\left(x - \frac{1}{x}\right)$$
$$= 8^3 + 3 \times 8$$
$$= 512 + 24$$
$$= \underline{\underline{536}}$$

16) Let $p(x) = px^4 + qx^3 + rx^2 + sx + u$
$x^2 - 1 = (x+1)(x-1)$
Since $x^2 - 1$ is a factor of $p(x)$, then $(x+1)$ and $(x-1)$ are also factors of $p(x)$.

Since $(x+1)$ is a factor of $p(x)$, then $p(-1)=0$

$\Rightarrow p(-1)^4+q(-1)^3+r(-1)^2+s(-1)+u=0$

$\Rightarrow p-q+r-s+u=0$

$\Rightarrow p+r+u=q+s \longrightarrow (1)$

Since $(x-1)$ is a factor of $p(x)$, then $p(1)=0$

$\Rightarrow p(1)^4+q(1)^3+r(1)^2+s(1)+u=0$

$\Rightarrow p+q+r+s+u=0$

$\Rightarrow (p+r+u)+q+s=0$

$\Rightarrow q+s+q+s=0$ [from eq:(1)]

$\Rightarrow 2(q+s)=0$

$\Rightarrow q+s=0 \longrightarrow (2)$

From (1) and (2), $\underline{\underline{p+r+u=q+s=0}}$

17) Since $f(y)$ is divided by $(y+1)$ the remainder $=10$, then $f(-1)=10$

$\Rightarrow (-1)^4-4(-1)^3+8(-1)^2-m(-1)+n=10$

$\Rightarrow 1+4+8+m+n=10$

$\Rightarrow m+n=10-13$

$\Rightarrow m+n=-3$

$\Rightarrow m=-3-n \longrightarrow (1)$

Since $f(y)$ is divided by $(y-1)$ the remainder $=16$, then $f(1)=16$

$\Rightarrow (1)^4-4(1)^3+8(1)^2-m(1)+n=16$

$\Rightarrow 1-4+8-m+n=16$

$\Rightarrow -m+n=16-5$

$\Rightarrow -m+n=11$

$\Rightarrow -(-3-n)+n=11$ [from eq:(1)]

$\Rightarrow 3+n+n=11$

$2n=11-3=8$

$n=\frac{8}{2}=\underline{\underline{4}}$

from eq:(1), $m=-3-4=\underline{\underline{-7}}$

$\therefore f(y)=y^4-4y^3+8y^2+7y+4$

Thus, remainder when $f(y)$ is divided by $(y-3)$ is $f(3)$.

$f(3) = 3^4 - 4(3)^3 + 8(3)^2 + 7\times 3 + 4$

$= 81 - 108 + 72 + 21 + 4$

$= 178 - 108 = \underline{\underline{70}}$

18) $(p+q)^3 - (p-q)^3 - 6q(p^2-q^2)$

$= p^3 + 3p^2q + 3pq^2 + q^3 - (p^3 - 3p^2q + 3pq^2 - q^3) - 6p^2q + 6q^3$

$= \cancel{p^3} + 3p^2q + \cancel{3pq^2} + q^3 - \cancel{p^3} + 3p^2q - \cancel{3pq^2} + q^3 - 6p^2q + 6q^3$

$= 6\cancel{p^2}q + 8q^3 - 6\cancel{p^2}q = \underline{\underline{8q^3}}$

19) $[a^3 - b^3 = (a-b)^3 + 3ab(a-b)]$

$27x^3 - 8y^3 = (3x)^3 - (2y)^3$; $a = 3x, b = 2y$

$= (3x-2y)^3 + 3\times 3x \times 2y(3x-2y)$

$= 13^3 + 18\times 5\times 13$

$= 2197 + 1170$

$= \underline{\underline{3367}}$

20) [If $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$]

(i) Checking:- $-14+8+6 = -14+14 = 0$

$\therefore (-14)^3 + 8^3 + 6^3 = 3\times -14\times 8\times 6$

$= \underline{\underline{-2016}}$

(ii) Checking:- $27-14-13 = 27-27 = 0$

$\therefore (27)^3 + (-14)^3 + (-13)^3 = 3\times 27\times -14\times -13$

$= \underline{\underline{14742}}$