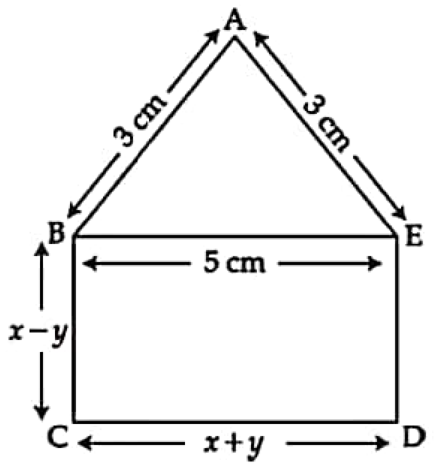
LINEAR EQUATIONS

1. Solve the following pair of linear equations.

   $3x+4y=10$ and $2x-2y=2$

2. The sum of the digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.

3. In the figure below ABCDE is a pentagon with BE || CD and BC || DE. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm, find the value of x and y.



4. Draw the graph of following equations :

   $2x+y=6$ and $4x-2y=4$.

   (i) Find the solution of equations from the graph.

   (ii) Find the area of triangle formed by these lines and x–axis.

5. For what value of k, $2x+3y=4$ and $(k+2)x+6y=3k+2$ will have infinitely many solutions.

6. 4 men and 6 boys can finish a piece of work in 5 days while 3 men and 4 boys can finish it in 7 days. Find the time taken by 1 man alone or that by 1 boy alone.

7. For what value of p will the following system of equations have no solution $(2p-1)\,x+(p-1)\,y=2p+1$; $y+3x-1=0$.

8. The sum of the numerator and denominator of a fraction is 12. If 1 is added to both the numerator and the denominator the fraction becomes $\frac{3}{4}$. Find the fraction.

9. One equation of a pair of dependent linear equations is $-5x+7y=2$, the second equation can be :

(A) $10x+14y+4=0$ (B) $-10x-14y+4=0$

(C) $-10x+14y+4=0$ (D) $10x-14y=-4$

10. Solve the equations graphically :

$2x+y=2$; $2y-x=4$

What is the area of the triangle formed by the two lines and the line $y=0$.

11. Solve $37x+43y=123$, $43x+37y=117$.

12. Solve:

$$x+\frac{6}{y}=6,\ 3x-\frac{8}{y}=5.$$

13. The taxi charges in a city consists of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. What are the fixed charges and the charges per km ?

14. The monthly incomes of A and B are in the ratio of 5 : 4 and their monthly expenditures are in the ratio of 7 : 5. If each saves Rs. 3000 per month. Find the monthly income of each.

15. Solve $148x+231y = 527$, $231x+148y=610$

16. The sum of the digits of a two digit number is 13. The number obtained by interchanging the digits of the given number exceeds that number by 27. Find the number.

17. Determine whether the following system of linear equations has a unique solution, no solution or infinitely many solutions :
$4x-5y=3$ and $8x-10y=6$.

18. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

19. Rekha's mother is five times as old as her daughter Rekha. Five years later, Rekha's mother will be three times as old as her daughter Rekha. Find the present age of Rekha and her mother's age.

20. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

21. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference between the digit is 3, find the number.

22. A boat goes 24 km up stream and 28 km down stream in 6 hours. It goes 30 km up stream and 21 km down stream in 6 hours 30 minutes. Find the speed of the boat in still water.

23. A person travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes 6 hours 30 minutes. But if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the car and that of the train.

24. Determine *a* and *b* for which the following system of linear equations has infinite number of solutions $2x-(a-4)y=2b+1$; $4x-(a-1)y=5b-1$.

25. Six years hence a man's age will be three times his son's age and three years ago, he was nine times as old as his son. Find their present ages.

26. Solve for x and y : $mx-ny=m^2+n^2$; $x-y=2n$

27. Find values of a and b for which the system of linear equations has infinite number of solutions :

$(a+b)x-2by=5a+2b+1$; $3x-y=14$

28. The sum of a 2 digit number and number obtained by reversing the order of digits is 99. If the digits of the number differ by 3. Find the number.

29. A sailor goes 8 km downstream in 40 minutes and returns in 1 hour. Find the speed of sailor in still water and the speed of current.

30. For which value of *k* will the following pair of linear equations have no solution.

$3x+y=1$; $(2k-1)\ x+(k-1)\ y=2k+1$

31. Solve for x and y :

$(a-b)\ x+(a+b)\ y=a^2-2ab-b^2$

$(a+b)\ (x+y)\ =\ a^2+b^2$

1.

Father's age is 3 times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

2.

Solve graphically ; $x-y=1$, $2x+y=8$. Shade the region bounded by these lines and y-axis. Also find its area.

3.

If $x=a$, $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are, respectively.

(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) -1 and -3

4.

For what value of k, will the system of linear equations $x+2y=5$ and $3x+ky-15=0$ has unique solution ?

5. Solve graphically the following equations :

$2x+y=6$; $-2y+3x=2$

Find the area of the triangle formed by the two lines and the y-axis.

6.

One equation of a pair of dependent linear equations is $-5x+7y=2$, the second equation can be :

(A) $10x+14y+4=0$ (B) $-10x-14y+4=0$

(C) $-10x+14y+4=0$ (D) $10x-14y=-4$

X Linear Equations

1) Solve the following pair of linear equations

$3x+4y=10$

$2x-2y=2$

Solution:-

$3x+4y=10 \rightarrow (1)$

$2x-2y=2 \rightarrow (2)$

$(1), \quad 3x+4y=10$

$(2)\times 2, \quad 4x-4y=4$

$(+), \quad 7x=14$

$x=2$

From eq: (1),

$6+4y=10$

$4y=4$

$y=1$

$$\boxed{\begin{array}{l} x=2 \\ y=1 \end{array}}$$

2) The sum of the digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.

Solution:-

Let the digit in the ten's place be x and that in the unit's place be y.

Original number $=10x+y$

Reversed number $=10y+x$

ATQ, $x+y=12 \rightarrow (1)$

Also, $(10y+x)-(10x+y)=18$

$\Rightarrow 10y+x-10x-y=18$

$\Rightarrow 9y-9x=18$

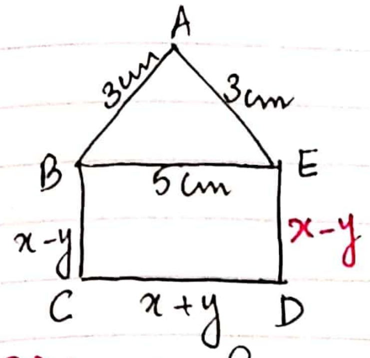
$\Rightarrow y-x=2 \longrightarrow (2)$

$(1)+(2), \quad 2y=14$

$y=7$

From eq: (1), $x=5$

$\therefore$ The required number is 57.

3) 

ABCDE is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. If the perimeter of ABCDE is 21cm, find the value of x and y.

Solution:- Since $BE \parallel CD$, $BC \parallel ED$ and $BC \perp CD$, BCDE is a rectangle.

Thus, $x+y = 5 \longrightarrow (1)$ [opposite sides of a rectangle]

Also, $BC = ED = x-y$.

Perimeter (pentagon ABCDE) $= 3+x-y+5+x-y+3 = 21$

$\Rightarrow 2x-2y = 21-11$

$\Rightarrow 2(x-y) = 10$

$x-y = 5 \longrightarrow (2)$

(1) + (2), $2x = 10$

$x = \underline{\underline{5}}$

From eq:(1), $y = \underline{\underline{0}}$

4) Draw the graph of following equations:
$2x+y=6$ and $4x-2y=4$
(i) Find the solution of equations from the graph
(ii) Find the area of Δ formed by these lines and x-axis.

Solution:-

$2x+y = 6$
$y = 6-2x$

x	0	3	2
y	6	0	2

$4x-2y = 4$
$4x-4 = 2y$
$y = \dfrac{4x-4}{2}$

x	0	1	2
y	-2	0	2

(i) From the graph, solution is (2, 2)
(ii) area of $\Delta = \frac{1}{2} \times b \times h = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times 2 \times 2 = \underline{\underline{2 \text{ sq. units}}}$

5) For what value of k, $2x+3y=4$ and $(k+2)x+6y=3k+2$ will have infinitely many solutions.

Solution:-

$2x+3y-4=0$

$(k+2)x+6y-(3k+2)=0$ are in the form $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$; where $a_1=2, b_1=3, c_1=-4$

$a_2=k+2, b_2=6, c_2=-(3k+2)$.

For infinitely many solutions, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$$\Rightarrow \underset{\text{I}}{\frac{2}{k+2}} = \underset{\text{II}}{\frac{3}{6}} = \underset{\text{III}}{\frac{-4}{-(3k+2)}}$$

From I and II, $12 = 3k+6$

$3k = 6$

$k = 2$

From II and III, $9k+6 = 24$

$9k = 18$

$k = 2$

Hence the required value of $k=2$

6) 4 men and 6 boys can finish a piece of work in 5 days while 3 men and 4 boys can finish it in 7 days. Find the time taken by 1 man alone or that by 1 boy alone.

Solution:-

Let the time taken by 1 man alone to complete the work be x days and that by 1 boy alone be y days

ATQ, $\frac{4}{x}+\frac{6}{y}=\frac{1}{5} \rightarrow (1)$

Also, $\frac{3}{x}+\frac{4}{y}=\frac{1}{7} \rightarrow (2)$

Put $\frac{1}{x}=a$; $\frac{1}{y}=b$

$4a+6b=\frac{1}{5} \xRightarrow{\times 3} 12a+18b=\frac{3}{5} \rightarrow (3)$

$3a+4b=\frac{1}{7} \xRightarrow{\times 4} 12a+16b=\frac{4}{7} \rightarrow (4)$

(3)−(4), $2b=\frac{3^{\times 7}}{5_{\times 7}}-\frac{4^{\times 5}}{7_{\times 5}}=\frac{21-20}{35}$

$b=\frac{1}{70}$

From eq: $4a+6b=\frac{1}{5}$

$$4a+\frac{6}{70}=\frac{1}{5}$$

$$4a=\frac{1^{\times 14}}{5_{\times 14}}-\frac{6}{70}=\frac{8}{70}$$

$$a=\frac{\cancel{8}^{2}}{{}_{1}\cancel{4}\times \cancel{70}_{35}}=\frac{1}{35}$$

$\therefore x=35$

$y=70$

Hence time taken by 1 man alone = 35 days

and by 1 boy alone = 70 days.

7) For what value of p will the following system of equations have no solution

$(2p-1)x+(p-1)y=2p+1$; $y+3x-1=0$

Solution:-

Let the given eq:s $(2p-1)x+(p-1)y-(2p+1)=0$

$3x+y-1=0$

be of the form $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$;

where $a_1=2p-1$, $b_1=p-1$, $c_1=-(2p+1)$

$a_2=3$, $b_2=1$, $c_2=-1$

For no solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$$\Rightarrow \underset{\mathrm{I}}{\frac{2p-1}{3}}=\underset{\mathrm{II}}{\frac{p-1}{1}}\neq\underset{\mathrm{III}}{\frac{\cancel{-}(2p+1)}{\cancel{-}1}}$$

From I and II, $2p-1=3p-3$

$-p=-2$

$p=2$

From II and III,

$p-1\neq 2p+1$

$-p\neq 2$

$p\neq -2$

8) The sum of numerator and denominator of a fraction is 12. If 1 is added to both the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction

Solution:-

Let the fraction be $\frac{x}{y}$.

ATQ, $x+y=12 \rightarrow (1)$

Also, $\frac{x+1}{y+1} = \frac{3}{4} \Rightarrow 4x+4 = 3y+3$

$\Rightarrow 4x-3y = -1 \rightarrow (2)$

(1) × 3, $3x+3y = 36$

(2), $4x-3y=-1$

(+), $7x = 35$

$x = 5$

From eq: (1), $y = 7$

Hence the fraction is $\frac{5}{7}$

9) One equation of a pair of dependent linear equations is $-5x+7y=2$, the second equation can be:

(A) $10x+14y+4=0$ (B) $-10x-14y+4=0$ (C) $-10x+14y+4=0$

(D) $10x-14y=-4$

Solution:-

For dependent linear equations: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$-5x+7y=2 \Rightarrow 5x-7y=-2 \rightarrow (1)$

Thus the other equation is $10x-14y=-4$ (D)

10) Solve the equations graphically:

$2x+y=2$; $2y-x=4$. What is the area of the Δ formed by the two lines and the line $y=0$.

Solution:-

$2x+y=2 \Rightarrow y = 2-2x \rightarrow (1)$

x	0	1	-2
y	2	0	6

$2y-x=4 \Rightarrow y = \frac{x+4}{2} \rightarrow (2)$

x	0	2	-2
y	2	3	1

$y=0$ is the x-axis

Thus the area of Δ formed by the given lines and x-axis $= \frac{1}{2} \times b \times h = \frac{1}{2} \times AB \times CO = \frac{1}{2} \times 5 \times 2 = 5$ sq. units

11) Solve: $37x+43y=123$; $43x+37y=117$

Solution:-

$$37x + 43y = 123 \rightarrow (1)$$
$$43x + 37y = 117 \rightarrow (2)$$

(1)+(2), $80x + 80y = 240 \xRightarrow{\div 80} x + y = 3$

(1)−(2), $-6x + 6y = 6 \xRightarrow{\div 6} -x + y = 1$

(+), $2y = 4$

$y = 2$

$x = 1$

12) Solve : $x + \frac{6}{y} = 6$; $3x - \frac{8}{y} = 5$

Solution:-

Put $\frac{1}{y} = a$

Then, $x + 6a = 6 \xRightarrow{\times 3} 3x + 18a = 18 \rightarrow (1)$

Also, $3x - 8a = 5 \rightarrow (2)$

(1)−(2), $26a = 13$

$a = \frac{13}{26} = \frac{1}{2}$

$\therefore y = 2$

From eq: $x + \frac{6}{y} = 6 \Rightarrow x + \frac{6}{2} = 6$

$\Rightarrow x = 6 - 3$

$\Rightarrow x = 3$

13) A taxi charges in a city consists of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charges per km?

Solution:-

Let the fixed charges be ₹x and the charges per km be ₹y

ATQ, $x + 10y = 105 \rightarrow (1)$

$x + 15y = 155 \rightarrow (2)$

(1)−(2), $-5y = -50$

$y = 10$

From eq: (1), $x + 100 = 105 \Rightarrow x = 5$

Hence the fixed charges = ₹ 5

the charges per km = ₹10

14) The monthly incomes of A and B are in the ratio of 5:4 and their monthly expenditures are in the ratio of 7:5. If each saves Rs 3000 per month. Find the monthly income of each.

Solution:-

Let the monthly incomes of A and B be $5x$ and $4x$ and monthly expenditures be $7y$ and $5y$ respectively.

Savings = income − expenditure

ATQ, $5x - 7y = 3000 \xRightarrow{\times 5} 25x - 35y = 15000$

Also, $4x - 5y = 3000 \xRightarrow{\times 7} \overset{(-)}{}28x \overset{(+)}{-} 35y \overset{(-)}{=} 21000$

$(-)$, $-3x = -6000$

$x = 2000$

From eq: $5x - 7y = 3000 \Rightarrow 10000 - 7y = 3000$

$-7y = -7000$

$y = 1000$

Hence the monthly income of A and B are $5 \times 2000 = ₹10,000$ and $4 \times 2000 = ₹8000$

15) Solve: $148x + 231y = 527$, $231x + 148y = 610$

Solution:-

$148x + 231y = 527 \rightarrow (1)$

$231x + 148y = 610 \rightarrow (2)$

(1)+(2), $379x + 379y = 1137$

$\div 379$, $x + y = 3 \rightarrow (1)$

(1)−(2), $-83x + 83y = -83$

$\div 83$, $-x + y = -1 \rightarrow (2)$

(1)+(2), $2y = 2$

$y = 1$

From eq: (1), $x = 2$

16) The sum of the digits of a two digit number is 13. The number obtained by interchanging the digits of the given number exceeds that number by 27. Find the number.

Solution:-

Let the digit in the ten's place be x and that in the unit's place be y.

ATQ, $x+y=13 \rightarrow (1)$

original number $= 10x+y$

Reversed number $= 10y+x$

Also, $(10y+x)-(10x+y)=27$

$\Rightarrow 10y+x-10x-y=27$

$\Rightarrow 9y-9x=27$

$\div 9$, $y-x=3 \rightarrow (2)$

$(1)+(2)$, $2y=16$

$y=8$

$x=5$

Hence the required number is 58

17) Determine whether the following system of linear equations has a unique solution, no solution or infinitely many solutions: $4x-5y=3$ and $8x-10y=6$

Solution:-

Let the given eq: $4x-5y-3=0$ and $8x-10y-6=0$ be of the form $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$; where $a_1=4$, $b_1=-5$, $c_1=-3$ and $a_2=8$, $b_2=-10$, $c_2=-6$

$\frac{a_1}{a_2}=\frac{4}{8}=\frac{1}{2}$

$\frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2}$

$\frac{c_1}{c_2}=\frac{-3}{-6}=\frac{1}{2}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Hence it has infinitely many solutions

18) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Solution:-

Let the no. of questions Yash answers correctly be x and that answered incorrectly be y.

Then, $3x-y=40 \xrightarrow{\times 2} 6x-2y=80 \rightarrow (1)$

$4x-2y=50 \rightarrow (2)$

$(1)-(2)$, $2x=30 \Rightarrow x=15$//

From eq: (2), $60 - 2y = 50$

$\Rightarrow -2y = -10$

$y = 5$

Hence the total no. of questions $= x+y = 15+5 = \underline{20}$

19) Rekha's mother is five times as old as her daughter Rekha. Five years later, Rekha's mother will be three times as old as her daughter Rekha. Find the present age of Rekha and her mother's age.

Solution:-

Let the present age of Rekha be x years and that of mother be y years.

ATQ, $y = 5x \longrightarrow (1)$

After 5 years, age of Rekha $= (x+5)$ yrs

age of mother $= (y+5)$ yrs.

Then, $y+5 = 3(x+5) \Rightarrow y+5 = 3x+15$

$\Rightarrow 3x - y = -10$

$\Rightarrow 3x - 5x = -10$ [from eq:(1)]

$-2x = -10$

$x = 5$

From eq: (1), $y = 25$

Hence the present ages of Rekha and her mother are 5 years and 25 years respectively.

20) Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4:5. Find the numbers.

Solution:-

Let the two numbers be $5x$ and $6x$.

ATQ, $\frac{5x-8}{6x-8} = \frac{4}{5} \Rightarrow 25x - 40 = 24x - 32$

$\Rightarrow 25x - 24x = -32 + 40$

$\Rightarrow x = 8$

Hence the two numbers are $5 \times 8 = \underline{40}$ and $6 \times 8 = \underline{48}$

21) Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference between the digit is 3, find the number.

Solution:-

Let the digit in the ten's place be x and digit in the unit's place be y.

Then, original number $= 10x+y$

reversed number $= 10y+x$.

ATQ, $7(10x+y) = 4(10y+x) \longrightarrow (1)$

Also, $y-x = 3 \longrightarrow (2)$

From eq: (1), $70x + 7y = 40y + 4x$

$\Rightarrow 66x - 33y = 0$

$\div 33$, $2x - y = 0 \longrightarrow (3)$

(2), $y - x = 3$

(3), $\underline{-y + 2x = 0}$

$(+)$, $x = 3$

From eq: (2), $y - 3 = 3$

$y = 6$

Hence the number is 36

22) A boat goes 24 km upstream and 28 km downstream in 6 hours. It goes 30 km upstream and 21 km downstream in 6 hours 30 minutes. Find the speed of the boat in still water.

Solution:- Time = Distance / Speed

Let the speed of boat in still water be x km/hr and speed of stream be y km/hr

ATQ, $\frac{24}{x-y} + \frac{28}{x+y} = 6 \rightarrow (1)$

Also, $\frac{30}{x-y} + \frac{21}{x+y} = 6 + \frac{\cancel{30}^{1}}{\cancel{60}_{2}} = \frac{13}{2} \rightarrow (2)$

Put $\frac{1}{x-y} = a$; $\frac{1}{x+y} = b$

Then, $24a + 28b = 6 \xRightarrow{\div 4} 6a + 7b = \frac{6}{4} = \frac{3}{2} \longrightarrow (3)$

And $30a + 21b = \frac{13}{2} \xRightarrow{\div 3} 10a + 7b = \frac{13}{6} \longrightarrow (4)$

$(3)-(4)$, $-4a = \frac{3\times 3}{2\times 3} - \frac{13}{6} = \frac{-4}{6} = \frac{-2}{3}$

$\therefore a = \frac{1}{6}$

From eq: (3) $\frac{6}{6} + 7b = \frac{3}{2}$

$$\Rightarrow 7b = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\therefore b = \frac{1}{14}$$

$$x - y = 6$$
$$x + y = 14$$
$$(+),\ 2x = 20$$
$$x = 10$$

From eq: $x + y = 14 \Rightarrow y = 14 - 10 = 4$

Hence Speed of boat in still water = 10 km/hr.

23) A person travels 600 km partly by train and partly by car. If he covers 400 km by train and the rest by car, it takes 6 hours 30 minutes. But if he travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the car and that of the train.

Solution:-

Let the speed of car be x km/hr and that of train be y km/hr.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

600: t 400, c 200

ATQ, $\frac{400}{y} + \frac{200}{x} = 6 + \frac{\cancel{30}\,1}{\cancel{60}\,2} \rightarrow (1)$

600: t 200, c 400

Also, $\frac{200}{y} + \frac{400}{x} = 6\frac{1}{2} + \frac{1}{2} = 7 \rightarrow (2)$

Put $\frac{1}{x} = a$; $\frac{1}{y} = b$

Then, $400b + 200a = \frac{13}{2} \overset{\div 2}{\Rightarrow} 200b + 100a = \frac{13}{4} \rightarrow (3)$

And $200b + 400a = 7 \rightarrow (4)$

$(3) - (4)$, $-300a = \frac{13}{4} - 7 = \frac{13-28}{4} = \frac{-15}{4}$

$$a = \frac{\cancel{-}\cancel{15}\,1}{\cancel{-}\cancel{300}_{20} \times 4} = \frac{1}{80} \Rightarrow \boxed{x = 80}$$

from eq: (3), $200b + \frac{\cancel{100}^{5}}{\cancel{80}_{4}} = \frac{13}{4}$

$$200b = \frac{13}{4} - \frac{5}{4} = \frac{8}{4} = 2$$

$$\therefore b = \frac{2}{200} = \frac{1}{100} \Rightarrow \boxed{y = 100}$$

Hence speed of car = 80 km/hr
and speed of train = 100 km/hr.

24) Determine a and b for which the following system of linear equations has infinite number of solutions.
$2x - (a-4)y = 2b+1$; $4x - (a-1)y = 5b-1$.

Solution:-

Let the eq:s $2x - (a-4)y - (2b+1) = 0$
$4x - (a-1)y - (5b-1) = 0$ be of the form
$a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ where
$a_1 = 2$, $b_1 = -(a-4)$, $c_1 = -(2b+1)$
$a_2 = 4$, $b_2 = -(a-1)$, $c_2 = -(5b-1)$
For infinite number of solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \underset{\mathrm{I}}{\frac{2}{4}} = \underset{\mathrm{II}}{\frac{\cancel{-}(a-4)}{\cancel{-}(a-1)}} = \underset{\mathrm{III}}{\frac{\cancel{-}(2b+1)}{\cancel{-}(5b-1)}}$$

From I and III, $\frac{2}{4} = \frac{2b+1}{5b-1} \Rightarrow 10b - 2 = 8b + 4$
$\Rightarrow 2b = 6$
$b = 3$

From I and II, $\frac{2}{4} = \frac{a-4}{a-1} \Rightarrow 2a-2 = 4a-16$

$\Rightarrow -2a = -14$

$a = 7$

$$\boxed{\begin{array}{l} a = 7 \\ b = 3 \end{array}}$$

25) Six years hence a man's age will be three times his son's age and three years ago, he was nine times as old as his son. Find their present ages.

Solution:-

Let the present ages of man and son be x years and y years respectively.

After 6 years, man's age $= x+6$

son's age $= y+6$

ATQ, $x+6 = 3(y+6)$

$\Rightarrow x+6 = 3y+18$

$\Rightarrow x-3y = 12 \rightarrow (1)$

3 years ago, man's age $= x-3$

son's age $= y-3$

ATQ, $x-3 = 9(y-3)$

$\Rightarrow x-3 = 9y-27$

$\Rightarrow x-9y = -24 \longrightarrow (2)$

(1) – (2), $6y = 36$

$y = 6$

From eq: (1), $x - 18 = 12$

$x = 30$

Hence present ages of man and his son are 30 years and 6 years respectively.

26) Solve for x and y : $mx - ny = m^2 + n^2$; $x - y = 2n$

Solution:-

Using cross multiplication method :

$$mx - ny - (m^2+n^2) = 0$$
$$x - y - 2n = 0$$

$$\begin{array}{ccc} m & -n & -(m^2+n^2) \\ 1 & -1 & -2n \end{array}$$

$$\frac{x}{\begin{array}{cc} -n & -(m^2+n^2) \\ -1 & -2n \end{array}} = \frac{y}{\begin{array}{cc} -(m^2+n^2) & m \\ -2n & 1 \end{array}} = \frac{1}{\begin{array}{cc} m & -n \\ 1 & -1 \end{array}}$$

$$\frac{x}{2n^2 - (m^2+n^2)} = \frac{y}{-(m^2+n^2) + 2mn} = \frac{1}{-m+n}$$

$$\frac{x}{2n^2-m^2-n^2} = \frac{y}{-m^2-n^2+2mn} = \frac{1}{-m+n}$$

$$\frac{x}{n^2-m^2} = \frac{y}{-(m^2+n^2-2mn)} = \frac{1}{n-m}$$

$$\therefore x = \frac{n^2-m^2}{n-m} = \frac{\cancel{(n-m)}(n+m)}{\cancel{n-m}} = \underline{\underline{n+m}}$$

$$y = \frac{-(m-n)^2}{n-m} = \frac{(m-n)^{\cancel{2}}}{\cancel{m-n}} = \underline{\underline{m-n}}$$

27) Find values of a and b for which the system of linear equations has infinite no. of solutions:
$(a+b)x - 2by = 5a + 2b + 1$; $3x - y = 14$

Solution:-

Let the given eq:s $(a+b)x - 2by - (5a+2b+1) = 0$ and $3x - y - 14 = 0$ be of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$; where $a_1 = (a+b)$, $b_1 = -2b$, $c_1 = -(5a+2b+1)$
$a_2 = 3$, $b_2 = -1$, $c_2 = -14$.

For infinite no. of solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \underset{\mathrm{I}}{\frac{a+b}{3}} = \underset{\mathrm{II}}{\frac{\cancel{-}2b}{\cancel{-}1}} = \underset{\mathrm{III}}{\frac{\cancel{-}(5a+2b+1)}{\cancel{-}14}}$$

From I and II, $a + b = 6b$
$a - 5b = 0 \longrightarrow (1)$

From II and III, $28b = 5a + 2b + 1$
$\Rightarrow 5a - 26b = -1 \rightarrow (2)$

$(1) \times 5$, $5a - 25b = 0$
(2), $\overset{(-)}{5a} \overset{(+)}{-} 26b \overset{(+)}{=} -1$
$(-)$, $b = 1$

From eq: (1), $a - 5 = 0$
$a = 5$

$\therefore \boxed{\begin{matrix} a = 5 \\ b = 1 \end{matrix}}$

28) The sum of a 2 digit number and number obtained by reversing the order of digits is 99. If the digits of the number differ by 3, find the number.

Solution:-

Let the digit in the ten's place be x and that in unit's place be y.

Original number $= 10x+y$

reversed number $= 10y+x$

ATQ, $10x+y+10y+x = 99$

$\Rightarrow 11x+11y = 99$

$(\div 11) \Rightarrow x+y = 9 \rightarrow (1)$

Also, $x-y = 3 \rightarrow (2)$

(1)+(2), $2x = 12$

$x = 6$

From eq: (1), $y = 3$

Hence the required number is 63

29) A sailor goes 8 km downstream in 40 mins and returns in 1 hour. Find the speed of the boat in still water and the speed of current.

Solution:-

Let the speed of the boat in still water be x km/hr and speed of current be y km/hr.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

ATQ, $\dfrac{8}{x+y} = \dfrac{\cancel{40}^{2}}{\cancel{60}_{3}} \Rightarrow \cancel{24}^{12} = \cancel{2}^{1}(x+y)$

$\Rightarrow x+y = 12 \rightarrow (1)$

$\dfrac{8}{x-y} = 1 \Rightarrow x-y = 8 \rightarrow (2)$

(1)+(2), $2x = 20$

$x = 10$

$y = 2$

Hence the speed of the boat in still water $= 10$ km/hr.

Speed of current $= 2$ km/hr.

30) For which value of k will the following pair of linear equation have no solution:

$3x+y = 1$; $(2k-1)x+(k-1)y = 2k+1$

Solution :-

Let the given equations $3x+y-1=0$; $(2k-1)x+(k-1)y-(2k+1)=0$ be of the form $a_1x+b_1y+c_1=0$; $a_2x+b_2y+c_2=0$; where

$a_1=3$, $b_1=1$, $c_1=-1$

$a_2=2k-1$, $b_2=k-1$, $c_2=-(2k+1)$

For no solution, $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}\neq\dfrac{c_1}{c_2}$

$$\Rightarrow \underset{\text{I}}{\frac{3}{2k-1}} = \underset{\text{II}}{\frac{1}{k-1}} \neq \underset{\text{III}}{\frac{\cancel{-}1}{\cancel{-}(2k+1)}}$$

From I and II, $3k-3=2k-1$

$$\Rightarrow \underline{\underline{k=2}}$$

From II and III, $2k+1\neq k-1$

$$\underline{\underline{k\neq -2}}$$

31) Solve for x and y :

$(a-b)x+(a+b)y=a^2-2ab-b^2$

$(a+b)(x+y)=a^2+b^2$

Solution:-

$$(a-b)x+(a+b)y=a^2-2ab-b^2 \longrightarrow (1)$$

$$\overset{(-)}{(a+b)x}\overset{(-)}{+}(a+b)y\overset{(-)}{=}(a^2+b^2) \longrightarrow (2)$$

$(1)-(2)$, $(\cancel{a}-b\cancel{-a}-b)x = \cancel{a^2}-2ab-b^2\cancel{-a^2}-b^2$

$$-2bx=-2b^2-2ab$$

$$\cancel{-2b}x=\cancel{-2b}(b+a)$$

$$\therefore x=a+b //$$

From eq: (1), $(a-b)(a+b)+(a+b)y=a^2-2ab-b^2$

$$\Rightarrow a^2-b^2+(a+b)y=a^2-2ab-b^2$$

$$\Rightarrow (a+b)y=\cancel{a^2}-2ab\cancel{-b^2}\cancel{-a^2}\cancel{+b^2}$$

$$\underline{\underline{y=-\frac{2ab}{a+b}}}$$

32) Father's age is 3 times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Solution :-

Let the father's age be x years and sum of ages of his two children be y years.

ATQ, $x = 3y \longrightarrow (1)$

After 5 years, father's age $= x+5$

2 children's age $= y+5+5 = y+10$

Also, $x+5 = 2(y+10)$

$\Rightarrow x+5 = 2y+20$

$\Rightarrow x-2y = 15$

$\Rightarrow 3y-2y = 15$ [from eq:(1)]

$y = 15$

From eq:(1), $x = 3\times15 = 45$

Hence age of father $= 45$ years.

33) If $x = a, y = b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are respt.

(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) -1 and -3

Solution:-

when $x=a, y=b$; $a-b=2$

$a+b=4$

$(+)$, $2a=6$

$a=3$

$b=1$ (C) 3 and 1

34) For what value of k will system of linear equations $x+2y=5$ and $3x+ky-15=0$ has unique solution?

Solution:-

Let the given eq:s $x+2y-5=0$ and $3x+ky-15=0$ are of the form $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$; where $a_1=1, b_1=2, c_1=-5$; $a_2=3, b_2=k, c_2=-15$.

For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{3} \neq \frac{2}{k}$

$\Rightarrow k \neq 6$

ie, k can take any real values other than 6.

35) Solve graphically the following equations: $2x+y=6$; $-2y+3x=2$. Find the area of the Δ formed

by the two lines and the y-axis.

36) One equation of a pair of dependent linear equations is $-5x+7y=2$, the second equation can be:

(A) $10x+14y+4=0$ (B) $-10x-14y+4=0$

(C) $-10x+14y+4=0$ (D) $10x-14y=-4$

Solution :-

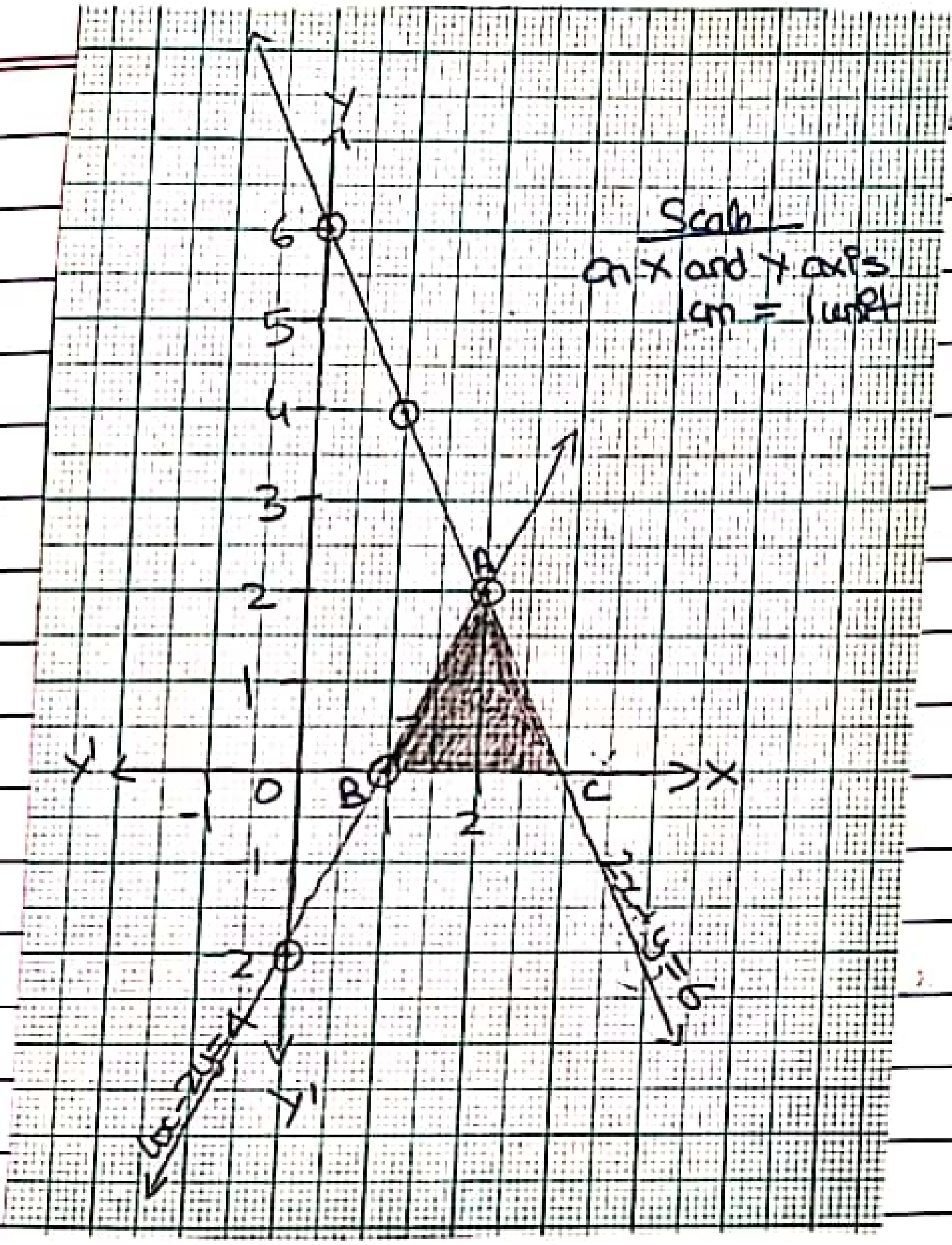
For dependent linear equations $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$$-5x+7y=2 \Rightarrow 5x-7y=-2$$

$$(\times 2) \quad 10x-14y=-4 \ \text{(D)}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{2}$$

7) For ...

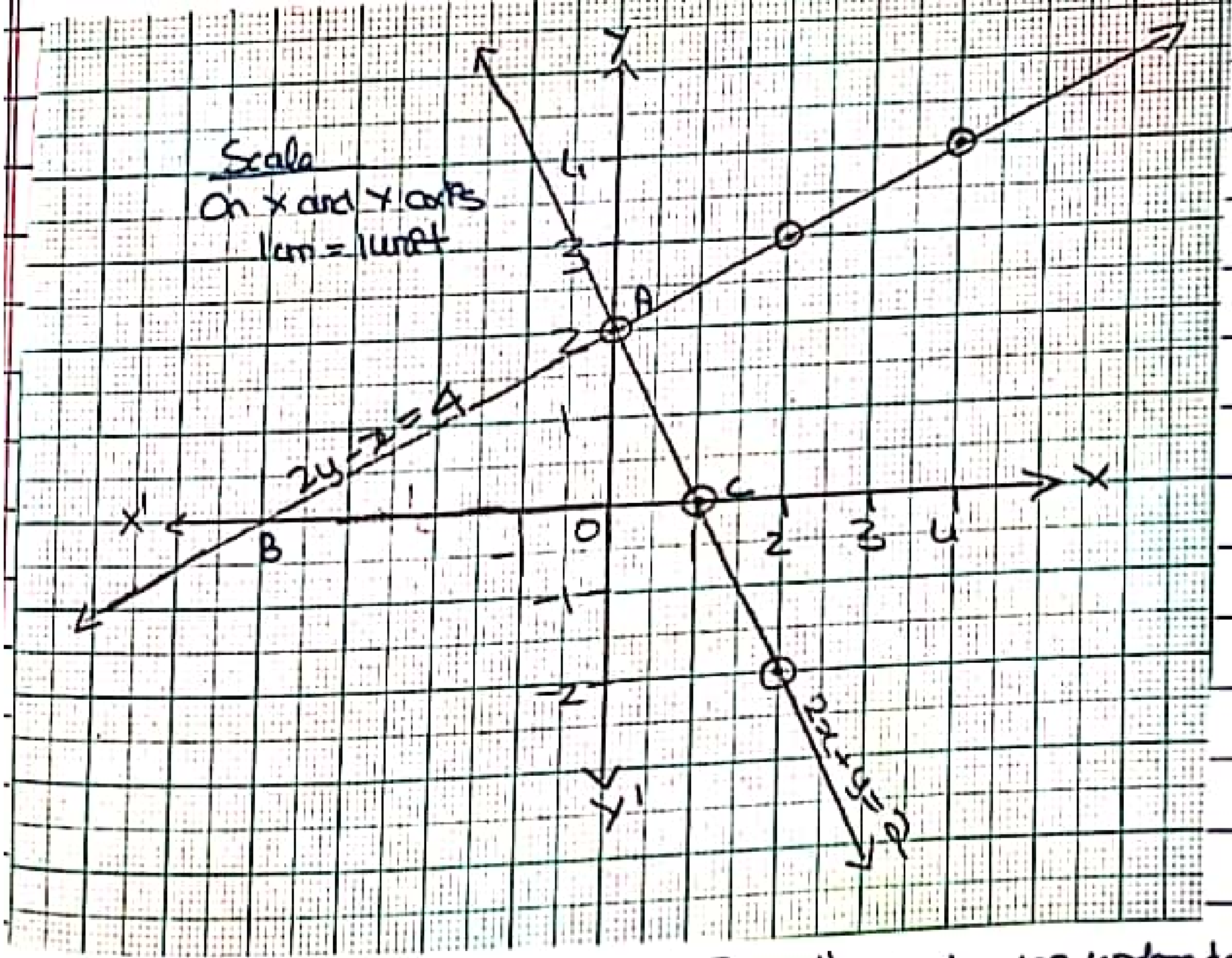


i) From the graph we get to know that the solution is (2,2) i.e, the intersecting point

$\therefore x = 2$

$\therefore y = 2$

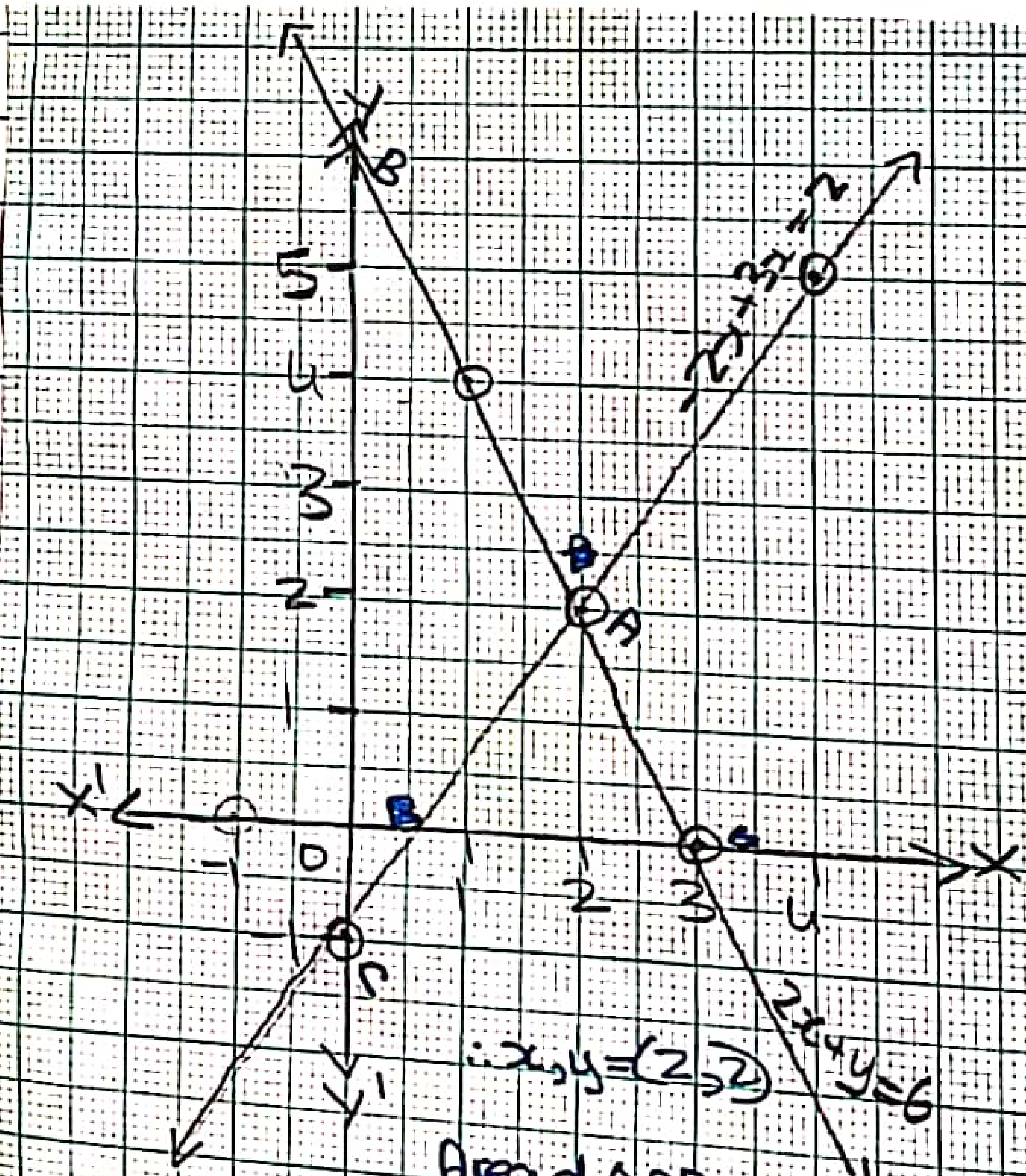
7/2



From the graph we understand that the intersecting point is the solution

$\therefore (0, 2)$ is the solution

$\boxed{\therefore x = 0 \quad \therefore y = 2}$



$\therefore x, y = (2, 2)$

Area of $\Delta ABC = \frac{1}{2} \times B \times H$

$= \frac{1}{\cancel{2}} \times 7 \times \cancel{2}$

$= 7 \text{ cm}^2$