IX Elite work – 5 (Algebraic Identities / RDS solutions)

1) If $x + \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2} =$

(a) 25 (b) 10 (c) 23 (d) 27

Solution:-

$$\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \qquad [(a+b)^2 = a^2+b^2+2ab]$$

$$5^2 = x^2 + \frac{1}{x^2} + 2$$

$$\therefore x^2 + \frac{1}{x^2} = 25 - 2 = 23 \text{ (c)}$$

2) If $x + \frac{1}{x} = 2$, then $x^3 + \frac{1}{x^3} =$

(a) 64 (b) 14 (c) 8 (d) 2

Solution:-

$$[a^3+b^3 = (a+b)^3 - 3ab(a+b)]$$

$$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$$

$$= 2^3 - 3\times 2$$

$$= 8 - 6 = 2 \text{ (d)}$$

3) If $x + \frac{1}{x} = 4$, then $x^4 + \frac{1}{x^4} =$

(a) 196 (b) 194 (c) 192 (d) 190

Solution:-

$$[a^2+b^2 = (a+b)^2 - 2ab]$$

$$x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

$$= 4^2 - 2 = 16 - 2 = 14$$

Now, $\left(x^2+\frac{1}{x^2}\right)^2 = 14^2$

$$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 196$$

$$\Rightarrow x^4 + \frac{1}{x^4} = 196 - 2 = 194 \text{ (b)}$$

4) If $x + \frac{1}{x} = 3$, then $x^6 + \frac{1}{x^6} =$

(a) 927 (b) 414 (c) 364 (d) 322

Solution:- $[a^2+b^2=(a+b)^2-2ab]$

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=3^2-2=9-2=7$$

$[a^3+b^3=(a+b)^3-3ab(a+b)]$

$$x^6+\frac{1}{x^6}=(x^2)^3+\frac{1}{(x^2)^3}$$

$$=\left(x^2+\frac{1}{x^2}\right)^3-3\left(x^2+\frac{1}{x^2}\right)$$

$$=7^3-3\times7=343-21$$

$$=\underline{\underline{322}}\ (d)$$

5) If $x^2+\frac{1}{x^2}=102$, then $x-\frac{1}{x}=$

(a) 8 (b) 10 (c) 12 (d) 13

Solution:-

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2 \qquad [(a-b)^2=a^2+b^2-2ab]$$

$$=102-2=100$$

$$\therefore\ x-\frac{1}{x}=\sqrt{100}=10\ (b)$$

6) If $x^3+\frac{1}{x^3}=110$, then $x+\frac{1}{x}=$

(a) 5 (b) 10 (c) 15 (d) none of these

Solution:-

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^3=110+3\left(x+\frac{1}{x}\right)$$

Let $x+\frac{1}{x}=y$

Then $y^3=110+3y$

$$\Rightarrow y^3-3y-110=0$$

When $y=5$, $5^3 - 3\times5 - 110 = 125-15-110 = 110-110=0$

$\therefore x+\frac{1}{x} = 5$ (a)

7) If $x^3 - \frac{1}{x^3} = 14$, then $x+\frac{1}{x} =$

(a) 5 (b) 4 (c) 3 (d) 2

Solution :-

$[a^3 - b^3 = (a-b)^3 + 3ab(a-b)]$

$$x^3 - \frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$$

$$14 = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$$

Let $x - \frac{1}{x} = y$

Then, $14 = y^3 + 3y$

$\Rightarrow y^3+3y-14=0$

When $y=2$, $2^3+3\times2-14 = 8+6-14 = 14-14=0$

$\therefore x-\frac{1}{x} = 2$ (d)

8) If $a+b+c=9$ and $ab+bc+ca=23$, then $a^2+b^2+c^2=$

(a) 35 (b) 58 (c) 127 (d) none of these

Solution:-

$[(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)]$

$$9^2 = a^2+b^2+c^2+2\times23$$

$$81-46 = a^2+b^2+c^2$$

$\therefore a^2+b^2+c^2 = 35$ (a)

9) $(a-b)^3+(b-c)^3+(c-a)^3 =$

(a) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ (b) $(a-b)(b-c)(c-a)$

(c) $3(a-b)(b-c)(c-a)$ (d) none of these

Solution:- [If $x+y+z=0$, then $x^3+y^3+z^3 = 3xyz$]

Checking:- $\cancel{a}-\cancel{b}+\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a}=0$

Then $(a-b)^3+(b-c)^3+(c-a)^3 = 3(a-b)(b-c)(c-a)$ (c)

10) If $\frac{a}{b}+\frac{b}{a} = -1$, then $a^3-b^3=$

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) 0

Solution:- $\dfrac{a^{xa}}{b^{xa}} + \dfrac{b^{xb}}{a^{xb}} = -1$

$$\Rightarrow \frac{a^2+b^2}{ab} = -1$$

$$\Rightarrow a^2+b^2 = -ab$$

$$\Rightarrow a^2+b^2+ab = 0$$

$$\therefore a^3-b^3 = (a-b)(a^2+b^2+ab)$$
$$= (a-b)\times 0$$
$$= 0 \text{ (d)}$$

11) If $a-b = -8$ and $ab = -12$, then $a^3-b^3 =$
(a) -244 (b) -240 (c) -224 (d) -260

Solution:-

$$a^3-b^3 = (a-b)^3 + 3ab(a-b)$$
$$= (-8)^3 + 3\times(-12)\times(-8)$$
$$= -512+288 = -224 \text{ (c)}$$

12) If the volume of a cuboid is $3x^2-27$, then its possible dimensions are
(a) $3, x^2, -27x$ (b) $3, x-3, x+3$ (c) $3, x^2, 27x$ (d) $3, 3, 3$

Solution:-

Volume of a cuboid $= l\times b\times h = 3x^2-27$
$= 3(x^2-9)$
$= 3(x+3)(x-3)$ $[a^2-b^2 = (a+b)(a-b)]$
(b)

13) $75\times75 + 2\times75\times25 + 25\times25$ is equal to
(a) 10000 (b) 6250 (c) 7500 (d) 3750

Solution :-

$75^2 + 2\times75\times25 + 25^2$ $[a^2+2ab+b^2 = (a+b)^2]$
$= (75+25)^2 = 100^2 = \underline{\underline{10000}}$ (a)

14) $(x-y)(x+y)(x^2+y^2)(x^4+y^4) =$

(a) $x^{16}-y^{16}$ (b) x^8-y^8 (c) x^8+y^8 (d) $x^{16}+y^{16}$

Solution :- $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$
$= (x^2-y^2)(x^2+y^2)(x^4+y^4)$ $[(a+b)(a-b) = a^2-b^2]$
$= [(x^2)^2-(y^2)^2][x^4+y^4] = (x^4-y^4)(x^4+y^4)$
$= (x^4)^2-(y^4)^2 = x^8-y^8$ (b)

15) If $x^4+\frac{1}{x^4} = 623$, then $x+\frac{1}{x}$

(a) 27 (b) 25 (c) $3\sqrt{3}$ (d) $-3\sqrt{3}$

Solution:-

$$\left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2 = 623+2 = 625$$

$$x^2+\frac{1}{x^2} = \sqrt{625} = 25$$

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 = 25+2 = 27$$

$$\therefore\ x+\frac{1}{x} = \sqrt{27} = 3\sqrt{3}\ \text{(c)}$$

$$\begin{array}{r|l} 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

16) If $x^4+\frac{1}{x^4} = 194$, then $x^3+\frac{1}{x^3} =$

(a) 76 (b) 52 (c) 64 (d) none of these

Solution:-

$$a^2+b^2 = (a+b)^2 - 2ab$$

$$x^4+\frac{1}{x^4} = \left(x^2+\frac{1}{x^2}\right)^2 - 2$$

$$194 = \left(x^2+\frac{1}{x^2}\right)^2 - 2$$

$$\therefore \left(x^2+\frac{1}{x^2}\right)^2 = 194+2 = 196$$

$$\therefore\ x^2+\frac{1}{x^2} = \sqrt{196} = 14$$

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 = 14+2 = 16$$

$$\therefore\ x+\frac{1}{x} = \sqrt{16} = 4$$

Thus, $x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$$= 4^3 - 3\times 4$$

$$= 64-12 = 52\ \text{(b)}$$

17) If $x-\frac{1}{x} = \frac{15}{4}$, then $x+\frac{1}{x} =$ (a) 4 (b) $\frac{17}{4}$ (c) $\frac{13}{4}$ (d) $\frac{1}{4}$

Solution:- $(a+b)^2 = (a-b)^2 + 4ab$

$$\left(x+\frac{1}{x}\right)^2 = \left(x-\frac{1}{x}\right)^2 + 4$$

$$= \left(\frac{15}{4}\right)^2 + 4$$

$$= \frac{225}{16} + 4 = \frac{225+64}{16} = \frac{289}{16}$$

$$\therefore x+\frac{1}{x} = \sqrt{\frac{289}{16}} = \frac{17}{4} \text{ (b)}$$

18) If $3x + \frac{2}{x} = 7$, then $9x^2 - \frac{4}{x^2} =$

(a) 25 (b) 35 (c) 49 (d) 30

Solution:-

$$(a-b)^2 = (a+b)^2 - 4ab$$

$$\left(3x - \frac{2}{x}\right)^2 = \left(3x + \frac{2}{x}\right)^2 - 4 \times 3\cancel{x} \times \frac{2}{\cancel{x}}$$

$$= 7^2 - 24 = 49 - 24 = 25$$

$$\therefore 3x - \frac{2}{x} = \sqrt{25} = 5$$

Then $9x^2 - \frac{4}{x^2} = \left(3x + \frac{2}{x}\right)\left(3x - \frac{2}{x}\right)$ $[a^2 - b^2 = (a+b)(a-b)]$

$$= 7 \times 5 = 35 \text{ (b)}$$

19) If $a^2+b^2+c^2-ab-bc-ca=0$, then

(a) $a+b=c$ (b) $b+c=a$ (c) $c+a=b$ (d) $a=b=c$

Solution:-

$$a^2+b^2+c^2-ab-bc-ca=0$$

$(\times 2)$ $2a^2+2b^2+2c^2-2ab-2bc-2ca=0$

$$\Rightarrow a^2+a^2+b^2+b^2+c^2+c^2-2ab-2bc-2ca=0$$

$$\Rightarrow (a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ca)=0$$

$$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$$

Thus $a-b=0 \Rightarrow a=b$

$b-c=0 \Rightarrow b=c$

$c-a=0 \Rightarrow c=a$

$$\therefore a=b=c \text{ (d)}$$

20) If $a+b+c=0$, then $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=$

(a) 0 (b) 1 (c) −1 (d) 3

Solution:-

If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Then, $\frac{a^{2}\times a}{a\times bc}+\frac{b^{2}\times b}{ca\times b}+\frac{c^{2}\times c}{ab\times c}=\frac{a^3+b^3+c^3}{abc}=\frac{3\cancel{abc}}{\cancel{abc}}=3$ (d)

21) If $a^{1/3}+b^{1/3}+c^{1/3}=0$, then

(a) $a+b+c=0$ (b) $(a+b+c)^3=27abc$ (c) $a+b+c=3abc$
(d) $a^3+b^3+c^3=0$

Solution:-

[If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$]

If $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}=0$, then $(a^{\frac{1}{3}})^3+(b^{\frac{1}{3}})^3+(c^{\frac{1}{3}})^3=3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$

$\Rightarrow a+b+c=3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$

$=3(abc)^{\frac{1}{3}}$

$\therefore (a+b+c)^3=\left(3(abc)^{\frac{1}{3}}\right)^3$ [Cubing on both sides]

$\Rightarrow (a+b+c)^3=27abc$ (b)

22) If $a+b+c=9$ and $ab+bc+ca=23$, then $a^3+b^3+c^3-3abc=$

(a) 108 (b) 207 (c) 669 (d) 729

Solution:-

$a+b+c=9$

$\Rightarrow (a+b+c)^2=9^2$ [Squaring on both sides]

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=81$

$\Rightarrow a^2+b^2+c^2+2\times 23=81$

$\Rightarrow a^2+b^2+c^2=81-46=35 \longrightarrow (1)$

$\therefore a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-(ab+bc+ca))$

$=9(35-23)$

$=9\times 12=108$ (a)

23) $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}=$

(a) $3(a+b)(b+c)(c+a)$ (b) $3(a-b)(b-c)(c-a)$
(c) $(a-b)(b-c)(c-a)$ (d) $(a+b)(b+c)(c+a)$

Solution:-

If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$

checking:-

$$\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}=0$$

$$\cancel{a}-\cancel{b}+\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a}=0$$

$$\therefore \frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}=\frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{(a-b)(b-c)(c-a)}$$

$$=\frac{\cancel{3}(a+b)\cancel{(a-b)}(b+c)\cancel{(b-c)}(c+a)\cancel{(c-a)}}{\cancel{3}\cancel{(a-b)}\cancel{(b-c)}\cancel{(c-a)}}$$

$$=(a+b)(b+c)(c+a) \quad (d)$$

24) The product of $(a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)=$
(a) a^6+b^6 (b) a^6-b^6 (c) a^3-b^3 (d) a^3+b^3

Solution:-

$$[(a+b)(a^2-ab+b^2)][(a-b)(a^2+ab+b^2)]$$

$$=(a^3+b^3)(a^3-b^3)$$

$$=(a^3)^2-(b^3)^2 \qquad [(x+y)(x-y)=x^2-y^2]$$

$$=a^6-b^6 \quad (b)$$

25) The product of $(x^2-1)(x^4+x^2+1)$ is equal to
(a) x^8-1 (b) x^8+1 (c) x^6-1 (d) x^6+1

Solution:-

$$[(a-b)(a^2+b^2+ab)=a^3-b^3]$$

$$(x^2-1)((x^2)^2+1^2+x^2\times 1)=(x^2)^3-1^3$$

$$=x^6-1 \quad (c)$$

26) If $\frac{a}{b}+\frac{b}{a}=1$, then $a^3+b^3=$

(a) 1 (b) -1 (c) $1/2$ (d) 0

Solution:-

$$\frac{a^{\times a}}{b\times a}+\frac{b^{\times b}}{a\times b}=1 \Rightarrow \frac{a^2+b^2}{ab}=1 \Rightarrow a^2+b^2=ab \Rightarrow a^2+b^2-ab=0 \rightarrow (1)$$

$\therefore\ a^3+b^3 = (a+b)(a^2+b^2-ab)$
$= (a+b)\times 0 = \underline{\underline{0}}$ (d)

27) If $49a^2 - b = \left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)$, then the value of b is

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

Solution :-

$$\left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right) = (7a)^2-\left(\frac{1}{2}\right)^2 \quad [(a+b)(a-b)=a^2-b^2]$$

$$= 49a^2 - \frac{1}{4}$$

On comparing, the value of b is $\frac{1}{4}$ (b)