IX Elite work-4

1) $\sqrt{10}\times\sqrt{15}$ is equal to (a) $5\sqrt{6}$ (b) $6\sqrt{5}$ (c) $\sqrt{30}$ (d) $\sqrt{25}$

Solution:-

$$\sqrt{10}\times\sqrt{15} = \sqrt{2}\times\sqrt{5}\times\sqrt{3}\times\sqrt{5} = 5\sqrt{6} \text{ (a)}$$

2) $\sqrt[5]{6}\times\sqrt[5]{6}$ is equal to

(a) $\sqrt[5]{36}$ (b) $\sqrt[5]{6\times 0}$ (c) $\sqrt[5]{6}$ (d) $\sqrt[5]{12}$

Solution:-

$$(6)^{\frac{1}{5}}\times(6)^{\frac{1}{5}} = (6\times 6)^{\frac{1}{5}} = (36)^{\frac{1}{5}} = \sqrt[5]{36} \text{ (a)}$$

3) The rationalisation factor of $\sqrt{3}$ is

(a) $-\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

Solution:-

$\frac{1}{\sqrt{3}}$ (b) [$\because$ rationalisation factor of $\sqrt{a}$ is $\frac{1}{\sqrt{a}}$]

4) The rationalisation factor of $2+\sqrt{3}$ is

(a) $2-\sqrt{3}$ (b) $\sqrt{2}+3$ (c) $\sqrt{2}-3$ (d) $\sqrt{3}-2$

Solution:-

$2-\sqrt{3}$ (a)

5) If $x=\sqrt{5}+2$, then $x-\frac{1}{x}$ equals

(a) $2\sqrt{5}$ (b) 4 (c) 2 (d) $\sqrt{5}$

Solution:-

$$\frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$$

$$\therefore x-\frac{1}{x} = (\sqrt{5}+2)-(\sqrt{5}-2) = \cancel{\sqrt{5}}+2-\cancel{\sqrt{5}}+2 = 4 \text{ (b)}$$

6) If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a-b\sqrt{3}$, then

(a) $a=2, b=1$ (b) $a=2, b=-1$ (c) $a=-2, b=1$ (d) $a=b=1$

Solution:-

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = \frac{\cancel{2}^{1}(2-\sqrt{3})}{\cancel{2}_{1}}$$

$$= 2-\sqrt{3}$$

$$\therefore a=2, b=1 \text{ (a)}$$

7) The simplest rationalising factor of $\sqrt[3]{500}$ is

(a) $\sqrt[3]{2}$ (b) $\sqrt[3]{5}$ (c) $\sqrt{3}$ (d) none of these

Solution:-

$\sqrt[3]{500} = \sqrt[3]{5\times5\times5\times2\times2}$

$= 5\times\sqrt[3]{4}$

5	500
5	100
5	20
2	4
	2

Thus the factor 2 is required to complete the triplet of 2.

$\therefore$ rationalising factor $= \sqrt[3]{2}$ (a)

8) The simplest rationalising factor of $\sqrt{3}+\sqrt{5}$ is
(a) $\sqrt{3}-5$ (b) $3-\sqrt{5}$ (c) $\sqrt{3}-\sqrt{5}$ (d) $\sqrt{3}+\sqrt{5}$

Solution:-

$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

$\therefore \sqrt{3}-\sqrt{5}$ (c)

9) The simplest rationalising factor of $2\sqrt{5}-\sqrt{3}$ is
(a) $2\sqrt{5}+3$ (b) $2\sqrt{5}+\sqrt{3}$ (c) $\sqrt{5}+\sqrt{3}$ (d) $\sqrt{5}-\sqrt{3}$

Solution:-

$(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3}) = 20-3 = 17$ (b)

10) If $x = \dfrac{2}{3+\sqrt{7}}$, then $(x-3)^2 =$ (a) 1 (b) 3 (c) 6 (d) 7

Solution:-

$$x = \frac{2(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{2(3-\sqrt{7})}{9-7} = \frac{2(3-\sqrt{7})}{2} = 3-\sqrt{7}$$

$$\therefore (x-3)^2 = (\cancel{3}-\sqrt{7}-\cancel{3})^2 = (-\sqrt{7})^2 = 7 \text{ (d)}$$

11) If $x = 7+4\sqrt{3}$ and $xy=1$, then $\dfrac{1}{x^2}+\dfrac{1}{y^2} =$

(a) 64 (b) 134 (c) 194 (d) 1/49

Solution:-

$$xy=1 \Rightarrow y = \frac{1}{x} = \frac{1}{7+4\sqrt{3}} = \frac{7-4\sqrt{3}}{(7)^2-(4\sqrt{3})^2}$$

$$= \frac{7-4\sqrt{3}}{49-48} = 7-4\sqrt{3}$$

$$\frac{1}{x^2}+\frac{1}{y^2} = \frac{y^2+x^2}{x^2y^2} = \frac{(x+y)^2-2xy}{(xy)^2}$$

$$= \frac{(7+\cancel{4\sqrt{3}}+7-\cancel{4\sqrt{3}})^2-2\times1}{(1)^2} = 14^2-2$$

$$= 196-2 = \underline{\underline{194}} \text{ (c)}$$

12) If $x+\sqrt{15}=4$, then $x+\frac{1}{x}=$
(a) 2 (b) 4 (c) 8 (d) 1

Solution:-

$$x = 4-\sqrt{15}$$

$$\frac{1}{x} = \frac{1}{4-\sqrt{15}} = \frac{4+\sqrt{15}}{4^2-(\sqrt{15})^2} = \frac{4+\sqrt{15}}{16-15} = 4+\sqrt{15}$$

$$\therefore x+\frac{1}{x} = 4-\cancel{\sqrt{15}}+4+\cancel{\sqrt{15}} = 8 \text{ (c)}$$

13) If $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then $x+y+xy=$
(a) 9 (b) 5 (c) 17 (d) 7

Solution:-

$$x = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{5+3+2\sqrt{15}}{5-3} = \frac{8+2\sqrt{15}}{2} = \frac{\cancel{2}^{1}(4+\sqrt{15})}{\cancel{2}\,1}$$
$$= 4+\sqrt{15}$$

$$y = \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{5+3-2\sqrt{15}}{5-3} = \frac{8-2\sqrt{15}}{2} = \frac{\cancel{2}^{1}(4-\sqrt{15})}{\cancel{2}\,1}$$
$$= 4-\sqrt{15}$$

$$xy = (4+\sqrt{15})(4-\sqrt{15}) = 4^2-(\sqrt{15})^2 = 16-15 = 1$$

$$\therefore x+y+xy = 4+\cancel{\sqrt{15}}+4-\cancel{\sqrt{15}}+1 = 9 \text{ (a)}$$

14) If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $x^2+xy+y^2=$
(a) 101 (b) 99 (c) 98 (d) 102

Solution:-

$$x = \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2-2\sqrt{6}}{3-2}$$
$$= 5-2\sqrt{6}$$

$$y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2+2\sqrt{6}}{3-2} = 5+2\sqrt{6}$$

$$xy = (5-2\sqrt{6})(5+2\sqrt{6}) = 5^2-(2\sqrt{6})^2 = 25-24 = 1$$

$$\therefore x^2+xy+y^2 = x^2+y^2+xy = (x+y)^2-2xy+xy$$
$$= (x+y)^2-xy = (5-\cancel{2\sqrt{6}}+5+\cancel{2\sqrt{6}})^2-1$$
$$= 10^2-1 = 100-1 =$$ [redacted] $$99 \text{ (b)}$$

15) $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(a) $3+2\sqrt{2}$ (b) $\dfrac{1}{3+2\sqrt{2}}$ (c) $3-2\sqrt{2}$ (d) $\dfrac{3}{2}-\sqrt{2}$

Solution:-

$$\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{3-2\sqrt{2}} = \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

$$= \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2} \text{ (a)}$$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

16) The value of $\dfrac{\sqrt{48}+\sqrt{32}}{\sqrt{27}+\sqrt{18}}$ is

(a) $\dfrac{4}{3}$ (b) 4 (c) 3 (d) $\dfrac{3}{4}$

Solution:-

$$\frac{4\sqrt{3}+4\sqrt{2}}{3\sqrt{3}+3\sqrt{2}}$$

$$= \frac{4\,\cancel{(\sqrt{3}+\sqrt{2})}^{1}}{3\,\cancel{(\sqrt{3}+\sqrt{2})}_{1}} = \frac{4}{3} \text{ (a)}$$

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \quad \begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array} \quad \begin{array}{r|l} 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \quad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

17) If $\dfrac{5-\sqrt{3}}{2+\sqrt{3}} = x+y\sqrt{3}$, then

(a) $x=13, y=-7$ (b) $x=-13, y=7$ (c) $x=-13, y=-7$ (d) $x=13, y=7$

Solution:-

$$\frac{5-\sqrt{3}}{2+\sqrt{3}} = \frac{(5-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{10-5\sqrt{3}-2\sqrt{3}+3}{2^2-(\sqrt{3})^2} = \frac{13-7\sqrt{3}}{4-3}$$

$$= 13-7\sqrt{3}$$

$\therefore x=13,\ y=-7$ (a)

18) If $x = \sqrt[3]{2+\sqrt{3}}$, then $x^3+\dfrac{1}{x^3} =$

(a) 2 (b) 4 (c) 8 (d) 9

Solution:-

$$x = \sqrt[3]{2+\sqrt{3}}$$

$$\Rightarrow x^3 = 2+\sqrt{3}$$

$$\therefore \frac{1}{x^3} = \frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

Then, $x^3+\dfrac{1}{x^3} = 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}} = 2+2 = 4$ (b)

19) The value of $\sqrt{3-2\sqrt{2}}$ is

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) $\sqrt{3}-\sqrt{2}$ (d) $\sqrt{3}+\sqrt{2}$

Solution:-

$$\sqrt{3-2\sqrt{2}} = \sqrt{2+1-2\times\sqrt{2}\times 1}$$

$$= \sqrt{(\sqrt{2})^2+(1)^2-2\times\sqrt{2}\times 1} = \sqrt{(\sqrt{2}-1)^2}$$

$$= \sqrt{2}-1 \text{ (a)}$$

20) The value of $\sqrt{5+2\sqrt{6}}$ is

(a) $\sqrt{3}-\sqrt{2}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $\sqrt{5}+\sqrt{6}$ (d) none of these

Solution:-

$$\sqrt{5+2\sqrt{6}} = \sqrt{3+2+2\times\sqrt{3}\times\sqrt{2}}$$

$$= \sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\times\sqrt{3}\times\sqrt{2}}$$

$$= \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{3}+\sqrt{2} \text{ (b)}$$

21) If $\sqrt{2} = 1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(a) 0.1718 (b) 5.8282 (c) 0.4142 (d) 2.4142

Solution:-

$$\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1^2}} = \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{(\sqrt{2}-1)^2}$$

$$= \sqrt{2}-1 = 1.4142-1 = 0.4142 \text{ (c)}$$

22) If $\sqrt{3}=1.732$ and $\sqrt{2} = 1.414$, then the value of $\sqrt{6}-\sqrt{3}$ upto three places of decimal is

(a) 0.235 (b) 0.717 (c) 1.414 (d) 0.471

Solution:-

$$\sqrt{6}-\sqrt{3} = \sqrt{2}\times\sqrt{3}-\sqrt{3} = \sqrt{3}(\sqrt{2}-1)$$

$$= 1.732(1.414-1)$$

$$= 1.732\times 0.414$$

$$= 0.717 \text{ (b)}$$

1732
414
6928
1732
6928
0.717048

23) The positive square root of $7+\sqrt{48}$ is

(a) $7+2\sqrt{3}$ (b) $7+\sqrt{3}$ (c) $2+\sqrt{3}$ (d) $3+\sqrt{2}$

Solution:-

$$7+\sqrt{48} = 7+4\sqrt{3}$$
$$= 4+3+2\times\sqrt{4}\times\sqrt{3}$$
$$= (\sqrt{4})^2+(\sqrt{3})^2+2\times\sqrt{4}\times\sqrt{3}$$
$$= (\sqrt{4}+\sqrt{3})^2$$

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$\therefore$ The positive square root is $\sqrt{4}+\sqrt{3} = 2+\sqrt{3}$ (c)

24) If $x=\sqrt{6}+\sqrt{5}$, then $x^2+\frac{1}{x^2}-2$

(a) $2\sqrt{6}$ (b) $2\sqrt{5}$ (c) 24 (d) 20

Solution:-

$$x = \sqrt{6}+\sqrt{5}$$

$$\frac{1}{x} = \frac{1}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} = \frac{\sqrt{6}-\sqrt{5}}{6-5} = \sqrt{6}-\sqrt{5}$$

$$x^2+\frac{1}{x^2}-2 = \left(x-\frac{1}{x}\right)^2 = \left[\sqrt{6}+\sqrt{5}-(\sqrt{6}-\sqrt{5})\right]^2$$
$$= \left(\cancel{\sqrt{6}}+\sqrt{5}-\cancel{\sqrt{6}}+\sqrt{5}\right)^2 = (2\sqrt{5})^2 = 4\times5$$
$$= 20 \text{ (d)}$$

25) If $\sqrt{13 - a\sqrt{10}} = \sqrt{8} + \sqrt{5}$, then $a =$

(a) -5 (b) -6 (c) -4 (d) -2

Solution :-

Squaring on both sides, $13 - a\sqrt{10} = (\sqrt{8} + \sqrt{5})^2$

$\Rightarrow 13 - a\sqrt{10} = 8 + 5 + 2 \times \sqrt{8} \times \sqrt{5}$

$\Rightarrow 13 - a\sqrt{10} = 13 + 2 \times 2\sqrt{2} \times \sqrt{5}$

$\Rightarrow \cancel{13} - a\sqrt{10} = \cancel{13} + 4\sqrt{10}$

$\therefore a = -4$ (c)