X Revision Worksheet (Homework-11)

Questions from Sample Papers (CBSE BOARD EXAM)

1) If $xy = 180$ and HCF $(x, y) = 3$, then find the LCM (x, y)
2) The decimal representation of $\frac{14587}{2^1 \times 5^4}$ will terminate after how many decimal places?
3) For what value of k, the pair of linear equations $3x + y = 3$ and $6x + ky = 8$ does not have a solution.
4) If 3 chairs and 1 table costs ₹1500 and 6 chairs and 1 table costs ₹2400 form linear equations to represent this situation.
5) Which term of the A.P 27, 24, 21, ... is zero?
6) In an A.P, if $d = -4$, $n = 7$, $a_n = 4$, then find a.
7) Find the roots of the equation $x^2 + 7x + 10 = 0$.
8) For what value (s) of 'a' quadratic equation $3ax^2 - 6x + 1 = 0$ has no real roots?
9) In the $\triangle ABC$, D and E are points on side AB and AC respectively such that $DE \parallel BC$. If $AE = 2cm$, $AD = 3cm$ and $BD = 4.5cm$, then find CE.
10) $\sin A + \cos B = 1$, $A = 30°$ and B is an acute angle, then find the value of B.

11) If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then find $x + y$.

12) 3 bells ring at an interval of 4, 7 and 14 minutes. All three bell rang at 6am, when the three balls will the ring together next?

13) Find a quadratic polynomial whose zeroes are $5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$.

14) If $\tan = \frac{3}{4}$, find the value of $\frac{1}{\sin A} + \frac{1}{\cos A}$

15) If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0° < \theta < 90°$, find the value of θ.

16) Prove that $2 - \sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational.

17) If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of p and the other root of the equation.

18) The roots α and β of the quadratic equation $x^2 - 5x + 3(k-1) = 0$ are such that $\alpha - \beta = 1$. find the value of k.

19) The perimeters of two similar triangles are 25cm and 15 cm respectively. If one side of the first triangle is 9cm, find the length of the corresponding side of the second triangle.

20) In an equilateral ΔABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

21) A motorboat covers a distance of 16km upstream and 24 km downstream in 6 hours. In the same time, it covers a distance of 12km upstream and 36 km downstream. find the speed of the boat in still water and that of the stream.

X Revision Worksheet (HW-11 answers)

1) $LCM(x,y) \times HCF(x,y) = x \times y$

$\Rightarrow LCM(x,y) \times 3 = 180$

$\therefore LCM(x,y) = \frac{180}{3} = \underline{\underline{60}}$

2) $\frac{14587}{2^1 \times 5^4}$, the decimal expansion terminates after four decimal places.

3) Let the given linear equations be of the form $a_1x+b_1y+c_1=0$
$a_2x+b_2y+c_2=0$

Where $a_1 = 3, b_1 = 1, c_1 = -3$
$a_2 = 6, b_2 = k, c_2 = -8$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{6} = \frac{1}{k} \neq \frac{\cancel{-}3}{\cancel{-}8}$

$\therefore \frac{3}{6} = \frac{1}{k} \Rightarrow k = \frac{6}{3} = \underline{\underline{2}}$

4) Let the cost of 1 chair be ₹x
and cost of 1 table be ₹y

Then, $3x+y = 1500 \Rightarrow 3x+y-1500 = 0 \rightarrow (1)$
$6x+y = 2400 \Rightarrow 6x+y-2400 = 0 \rightarrow (2)$

are the required linear equations in two variables.

5) $a = 27$
$d = 24-27 = -3$
$a_n = 0$
$n = ?$

$a_n = a+(n-1)d$
$0 = 27-3(n-1)$
$-27 = -3(n-1)$
$n-1 = \frac{-27}{-3} = 9$

$\therefore n = 9+1 = 10$

Hence 10^{th} term of the given A.P is 0.

6) $a_n = a+(n-1)d$

$\Rightarrow 4 = a+(7-1)\times -4$

$\Rightarrow 4 = a-28+4$

$\Rightarrow a-24 = 4$

$\therefore a = 4+24 = \underline{\underline{28}}$

7) $x^2+7x+10=0$

S	P	
7	10	$\langle$ 2, 5

$\Rightarrow (x+2)(x+5)=0$

$\therefore$ The roots are -5 and -2.

8) Let $3ax^2-6x+1=0$ be of the form $Ax^2+Bx+C=0$; where $A=3a$, $B=-6$, $C=1$

For no real roots, $B^2-4AC<0$

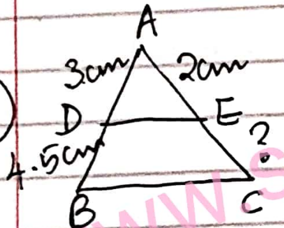
$36-4\times 3a\times 1<0$

$-12a<-36$

$12a>36$

$\therefore \underline{\underline{a>3}}$

9)



Since $DE \parallel BC$, using Thales theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{3}{4.5} = \frac{2}{EC}$$

$$\Rightarrow EC = \frac{2\times \cancel{4.5}^{1.5}}{\cancel{3}_{1}} = 1.5\times 2$$

$$= \underline{\underline{3cm}}$$

10) $\sin A + \cos B = 1$

$\sin 30^\circ + \cos B = 1$

$\frac{1}{2} + \cos B = 1$ $\quad [\because \sin 30^\circ = 1/2]$

$\therefore \cos B = 1 - \frac{1}{2} = \frac{1}{2}$

Thus, $B = 60^\circ$ $\quad [\because \cos 60^\circ = \frac{1}{2}]$

11) $x+y = 2\sin^2\theta + 2\cos^2\theta + 1$

$= 2(\sin^2\theta + \cos^2\theta) + 1$

$= 2+1 = \underline{3}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

12) LCM(4, 7, 14)

$4 = 2^2$

$7 = 7^1$

$14 = 2^1 \times 7^1$

$\therefore$ LCM(4, 7, 14) $= 2^2 \times 7 = 4 \times 7 = 28$ minutes

Hence all three bells ring together next at 6:28 am

13) Let $\alpha = 5 - 3\sqrt{2}$

$\beta = 5 + 3\sqrt{2}$

Then, $\alpha + \beta = 5 - \cancel{3\sqrt{2}} + 5 + \cancel{3\sqrt{2}} = 10$

$\alpha\beta = (5 - 3\sqrt{2})(5 + 3\sqrt{2}) = 5^2 - (3\sqrt{2})^2$

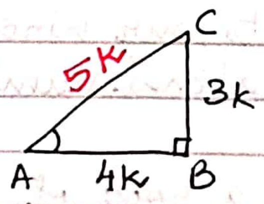
$= 25 - 18 = \underline{7}$

$\therefore$ The required quadratic polynomial

$= x^2 - (\alpha+\beta)x + \alpha\beta$

$= \underline{\underline{x^2 - 10x + 7}}$

14) $\tan A = \dfrac{3k}{4k} = \dfrac{BC}{AB}$



Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 16k^2 + 9k^2 = 25k^2$

$\therefore AC = \sqrt{25k^2} = 5k$

$\sin A = \dfrac{BC}{AC} = \dfrac{3k}{5k} = \dfrac{3}{5} \Rightarrow \dfrac{1}{\sin A} = \dfrac{5}{3}$

$\cos A = \dfrac{AB}{AC} = \dfrac{4k}{5k} = \dfrac{4}{5} \Rightarrow \dfrac{1}{\cos A} = \dfrac{5}{4}$

$\therefore \dfrac{1}{\sin A} + \dfrac{1}{\cos A} = \dfrac{5^{\times 4}}{3_{\times 4}} + \dfrac{5^{\times 3}}{4_{\times 3}} = \dfrac{20+15}{12} = \underline{\underline{\dfrac{35}{12}}}$

15) $\sqrt{3}\sin\theta - \cos\theta = 0$

$\Rightarrow \sqrt{3}\sin\theta = \cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\sqrt{3}}$

$\therefore \tan\theta = \dfrac{1}{\sqrt{3}}$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$\therefore \theta = 30^\circ$ $\left[\because \tan 30^\circ = \dfrac{1}{\sqrt{3}}\right]$

16) Let us assume that $2-\sqrt{3}$ is a rational number.

Then, $2-\sqrt{3} = \dfrac{a}{b}$; where a and b are integers and $b \neq 0$

$\Rightarrow -\sqrt{3} = \dfrac{a}{b} - 2$

$\Rightarrow \sqrt{3} = 2 - \dfrac{a}{b}$

$\Rightarrow \sqrt{3} = \dfrac{2b-a}{b}$, which is a rational number

Since a and b are integers. Thus $\sqrt{3}$ is also a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. The contradiction arises due to our wrong assumption that $2-\sqrt{3}$ is a rational number. Hence $2-\sqrt{3}$ is irrational.

17) Since $\frac{2}{3}$ is a root of $3x^2+px+4=0$, then

$3\left(\dfrac{2}{3}\right)^2 + p\left(\dfrac{2}{3}\right) + 4 = 0$

$\Rightarrow \cancel{3}^{1} \times \dfrac{4}{\cancel{9}_{3}} + \dfrac{2p}{3} + 4 = 0$

$\Rightarrow \dfrac{2p}{3} = -4 - \dfrac{4}{3} = \dfrac{-12-4}{3} = -\dfrac{16}{3}$

$\Rightarrow \dfrac{2p}{\cancel{3}} = \dfrac{-16}{\cancel{3}}$

$\Rightarrow p = -8$

Then $3x^2+px+4=0 \Rightarrow 3x^2-8x+4=0$

$\Rightarrow 3x^2-6x-2x+4=0$ S P -8 $12 <$ -6, -2

$\Rightarrow 3x(x-2)-2(x-2)=0$

$\Rightarrow (3x-2)(x-2)=0$

$\therefore x = \frac{2}{3}, 2$

Hence the other root $= 2$

18) Let the given equation be of the form $ax^2+bx+c=0$, where $a=1, b=-5, c=3(k-1)$

Then, $\alpha+\beta = -\frac{b}{a} = 5$

$\alpha\beta = \frac{c}{a} = 3(k-1)$

we know, $(a+b)^2 = (a-b)^2+4ab$

$\Rightarrow (\alpha+\beta)^2 = (\alpha-\beta)^2+4\alpha\beta$

$\Rightarrow 25 = 1+12(k-1)$

$\Rightarrow 25 = 1+12k-12$

$= 12k-11$

$\Rightarrow 12k = 25+11 = 36$

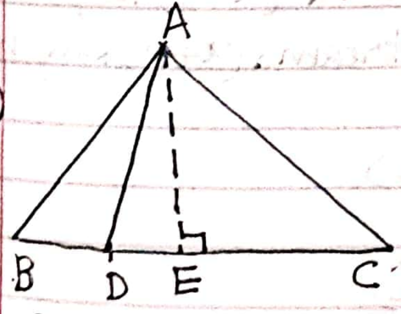
$\therefore k = \frac{36}{12} = 3$

19) we know that the ratio of perimeter of two similar triangles is equal to the ratio of their corresponding sides.

Thus, $\frac{25}{15} = \frac{9}{x}$ (let x be the corresponding side of second triangle)

$\Rightarrow x = \frac{9 \times \cancel{15}^{3}}{\cancel{25}_{5}}$

$= \frac{27}{5} = 5.4$ cm

20) 

Given: in equilateral $\triangle ABC$, $BD = \frac{1}{3}BC$

To prove: $9AD^2 = 7AB^2$

Construction: draw $AE \perp BC$

Proof :-

$BD = \frac{1}{3} BC \rightarrow (1)$

Since ΔABC is an equilateral Δ, AE is the perpendicular bisector of BC.

Thus, $BE = CE = \frac{1}{2} BC \rightarrow (2)$

Using Pythagoras Theorem;

in ΔADE, $AD^2 = AE^2 + DE^2$

$= AB^2 - BE^2 + DE^2$ ($\because$ in rt. ΔABE, $AB^2 = BE^2 + AE^2$)

$= AB^2 - BE^2 + (BE - BD)^2$

$= AB^2 - \cancel{BE^2} + \cancel{BE^2} + BD^2 - 2BE \cdot BD$

$= AB^2 + \left(\frac{1}{3} BC\right)^2 - \cancel{2} \times \frac{1}{\cancel{2}} BC \times \frac{1}{3} BC$ [from eq: (1) and (2)]

$= AB^2 + \frac{BC^2}{9} - \frac{BC^2}{3}$

$= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3}$ [$\because$ ABC is an equilateral Δ]

$= \frac{9AB^2 + AB^2 - 3AB^2}{9}$

$= \frac{7AB^2}{9}$

$\therefore 9AD^2 = 7AB^2$. Hence Proved.

21) Let the speed of the boat in still water be x km/hr and speed of stream be y km/hr.

Then, speed of boat upstream $= (x-y)$ km/hr

Speed of boat downstream $= (x+y)$ km/hr

ATQ, $\frac{16}{x-y} + \frac{24}{x+y} = 6 \longrightarrow (1)$

$\frac{12}{x-y} + \frac{36}{x+y} = 6 \longrightarrow (2)$

Put $\frac{1}{x-y} = a$; $\frac{1}{x+y} = b$

Then, $16a + 24b = 6 \overset{\div 4}{\Rightarrow} 4a + 6b = 6/4 \rightarrow (3)$

Also, $12a + 36b = 6 \overset{\div 3}{\Rightarrow} 4a + 12b = 2 \rightarrow (4)$

$(3)-(4)$, $-6b = \frac{\cancel{6}^{3}}{\cancel{4}_{2}} - 2$

$= \frac{3-4}{2}$

$\cancel{-}6b = \cancel{-}\frac{1}{2}$

$b = \frac{1}{12}$

From eq:(4), $4a + \frac{12}{12} = 2$

$4a = 2 - 1 = 1$

$a = \frac{1}{4}$

$\therefore x - y = 4$

$x + y = 12$

$(+)$, $2x = 16$

$x = \frac{16}{2} = 8$

$y = 4$

Hence the speed of boat in still water $= 8$ km/hr

Speed of stream $= 4$ km/hr