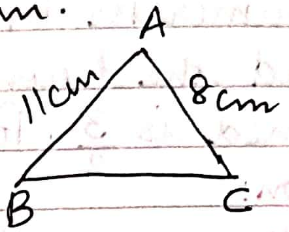
IX CHAPTER: 12 **HERON'S FORMULA**

Examples:

1) Find the area of a Δ, two sides of which are 8cm and 11cm and the perimeter is 32cm.



Solution:-

Perimeter $= AB + BC + AC = 32$

$\Rightarrow 11 + BC + 8 = 32$

$\Rightarrow BC = 32 - 19$

$\therefore BC = \underline{13\,cm}$

Let $a = 11cm, b = 13cm, c = 8cm$

$$S = \frac{a+b+c}{2} = \frac{11+13+8}{2} = \frac{32}{2} = \underline{\underline{16cm}}$$

Using Heron's formula,

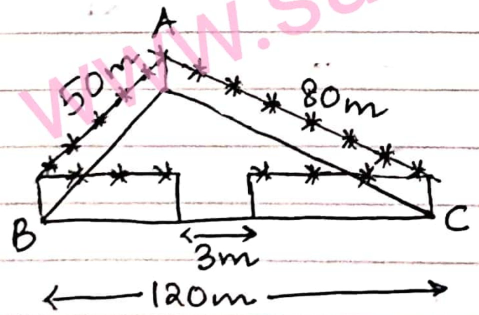
$$\text{Area}(\Delta ABC) = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{16(16-11)(16-13)(16-8)}$$

$$= \sqrt{16 \times 5 \times 3 \times \underset{4\times 2}{8}}$$

$$= 4 \times 2 \times \sqrt{5 \times 3 \times 2} = \underline{\underline{8\sqrt{30}\,cm^2}}$$

2)



A triangular park ABC has sides 120m, 80m and 50m. A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of Rs 20 per metre leaving a space 3m wide for a gate on one side.

Solution:-

Let $a = 50m, b = 80m, c = 120m$

$$S = \frac{a+b+c}{2} = \frac{50+80+120}{2} = \frac{250}{2} = 125m$$

Using Heron's formula,

area of triangular park $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{125(125-50)(125-80)(125-120)}$$

$$= \sqrt{125 \times 75 \times 45 \times 5}$$

$$\quad \underset{25\times5}{125} \quad \underset{25\times3}{75} \quad \underset{9\times5}{45} \quad \underset{5}{5}$$

$$= 25 \times 5 \times 3\sqrt{3\times5}$$

$$= 375\sqrt{15}\ m^2$$

Perimeter of the triangular park $= 50+80+120 = 250m$

length of wire needed for fencing $= 250-3 = 247m$

$\therefore$ Cost of fencing $=$ length of wire $\times$ rate

$$= 247 \times 20 = ₹\ 4940$$

3) The sides of a triangular plot are in the ratio 3:5:7 and its perimeter is 300m. Find its area.

Solution:-

Let the sides of triangular plot be $3x$, $5x$ and $7x$.

Then, perimeter $= 3x + 5x + 7x = 300$

$$\Rightarrow \quad 15x = 300$$

$$\therefore x = \frac{300}{15} = 20$$

Thus the sides are $3x = 3\times20 = 60m$

$5x = 5\times20 = 100m$

$7x = 7\times20 = 140m$

Let $a = 60m$, $b = 100m$, $c = 140m$.

Then, $s = \frac{a+b+c}{2} = \frac{60+100+140}{2} = \frac{300}{2} = 150m$

Using Heron's Formula,

area of triangular plot $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{150(150-60)(150-100)(150-140)}$$

$$= \sqrt{150 \times 90 \times 50 \times 10}$$

$$\quad \underset{50\times3}{150} \quad \underset{9\times10}{90} \quad \underset{50}{50} \quad \underset{10}{10}$$

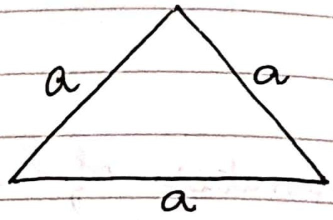
$$= 50 \times 3 \times 10\sqrt{3} = 1500\sqrt{3}\ m^2$$

Exercise 12.1

1) A traffic signal board, indicating 'SCHOOL AHEAD' is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180cm, what will be the area of the signal board?

Solution:-

$$S = \frac{a+a+a}{2} = \frac{3a}{2}$$



Using Heron's Formula,

$$\text{area of } \Delta = \sqrt{S(S-a)(S-a)(S-a)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$

$$= \sqrt{\frac{3}{16}a^4} = \frac{\sqrt{3}a^2}{4} \text{ square units}$$

Perimeter of equilateral triangular signal board

$$= 3a = 180$$

$$\Rightarrow a = \frac{180}{3} = 60 \text{ cm}$$

Area of signal board $= \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3} \times 60 \times \cancel{60}^{15}}{\cancel{4}_1}$

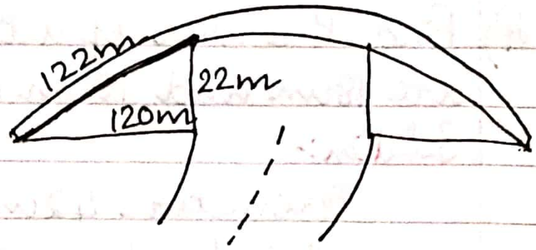
$$= 900\sqrt{3} \text{ cm}^2$$

2) The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122m, 22m and 120m. The advertisement yield an earning of ₹5000 per m² per year. A company hired one of its wall for 3 months. How much rent did it pay?

Solution:-

Let $a = 122$m; $b = 22$m; $c = 120$ m

$S = \frac{a+b+c}{2} = \frac{122+22+120}{2}$



$= \frac{264}{2} = 132m$

Using Heron's formula,
area of 1 triangular side wall =

$$\sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{132(132-122)(132-22)(132-120)}$$

$$= \sqrt{\underbrace{132}_{11\times12} \times \underbrace{10}_{10} \times \underbrace{110}_{11\times10} \times \underbrace{12}_{12}}$$

$$= 11 \times 12 \times 10 = 1320 m^2$$

$\therefore$ Rent paid for 3 months = area × rate

$$= \frac{\cancel{1320}^{330} \times 5000 \times \cancel{3}^{1}}{\cancel{12}\,\cancel{4}\,1}$$

$$= 330 \times 5000$$

$$= ₹\,1650000$$

3) There is a slide in a park. One of its side walls has been painted in some colour with a message 'KEEP THE PARK GREEN AND CLEAN. If the sides of the wall are 15m, 11m and 6m, then the area painted in colour.

Solution:-

Let $a = 11m, b = 15m, c = 6m$.

Then, $S = \frac{a+b+c}{2} = \frac{11+15+6}{2}$

$= \frac{32}{2} = 16m$

Using Heron's formula,
area painted in colour $= \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{16(16-15)(16-11)(16-6)}$$

$$= \sqrt{16 \times 1 \times 5 \times \underbrace{10}_{5\times2}} = 4 \times 5\sqrt{2}$$

$$= 20\sqrt{2}\, m^2$$

4) Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Solution:-

Perimeter = 42cm

$\therefore$ third side = $42 - (18+10) = 42 - 28 = 14$ cm //

Let $a = 18$cm, $b = 10$cm, $C = 14$ cm.

$$S = \frac{a+b+c}{2} = \frac{18+10+14}{2} = \frac{42}{2} = 21\text{cm} //$$

Thus, area of triangle $= \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{21(21-18)(21-10)(21-14)}$$

$$= \sqrt{\underset{3\times 7}{21} \times 3 \times 11 \times 7}$$

$$= 3 \times 7 \sqrt{11}$$

$$= 21\sqrt{11}\ \text{cm}^2$$

5) Sides of a Δ are in the ratio of 12:17:25 and its perimeter is 540cm. Find its area.

Solution:-

Let the sides be $12x$, $17x$ and $25x$.

Thus, perimeter = 540cm.

$\Rightarrow 12x + 17x + 25x = 540$

$\Rightarrow 54x = 540$

$$\therefore x = \frac{540}{54} = \underline{\underline{10}}$$

$\therefore$ The sides are $12x = 120$ cm,

$17x = 170$ cm

and $25x = 250$ cm.

Let $a = 120$cm, $b = 170$cm, $C = 250$cm; $S = \frac{\text{Perimeter}}{2} = \frac{540}{2} = 270$cm

Using Heron's formula,

area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

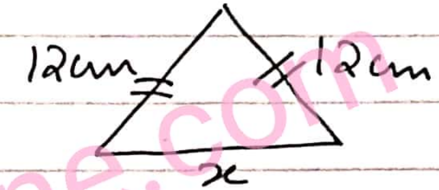
$$= \sqrt{270(270-120)(270-170)(270-250)}$$

$$= \sqrt{\underset{9\times 30}{270} \times \underset{30\times 5}{150} \times \underset{20\times 5}{100} \times \underset{20}{20}} = 3 \times 30 \times 5 \times 20$$

$$= 9000\ \text{cm}^2 //$$

6) An isosceles Δ has perimeter 30cm and each of the equal sides is 12cm. Find the area of the triangle.

Solution:-



Let x be the third side.

Then, $12 + 12 + x = 30$

$$x = 30 - 24 = 6\text{cm}$$

Thus the sides are 12cm, 12cm and 6cm.

Let $a = 12\text{cm}$, $b = 12\text{cm}$, $c = 6\text{cm}$.

Then, $s = \frac{a+b+c}{2} = \frac{12+12+6}{2} = \frac{30}{2} = 15\text{cm}$ //

Using Heron's formula,

area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{15(15-12)(15-12)(15-6)}$$

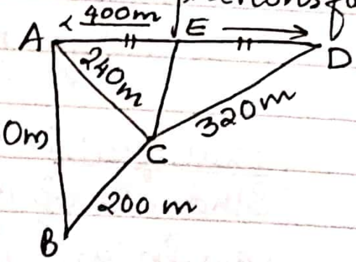
$$= \sqrt{\underset{3\times5}{15} \times 3 \times 3 \times 9}$$

$$= 3 \times 3\sqrt{15} = 9\sqrt{15}\text{cm}^2.$$

Application of Heron's formula in finding areas of quadrilateral

Example: 4

Kamla has a triangular field with sides 240m, 200m, 360 m where she grew wheat. In another triangular field with sides 240 m, 320 m, 400m adjacent to the previous field, she wanted to grow potatoes and onions. She divide the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew potatoes in one part and onions in the other part. How much area (in hectare) has been used for wheat, potatoes and onions? (1 hectare = 10,000 m^2)



Solution:-

For $\triangle ABC$, let $a = 360m$, $b = 200m$, $c = 240m$

$$S = \frac{a+b+c}{2} = \frac{360+200+240}{2} = \frac{800}{2} = 400m$$

Using Heron's formula, area($\triangle ABC$) $= \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{400(400-360)(400-200)(400-240)}$$

$$= \sqrt{\underbrace{400}_{4\times100}\times\underbrace{40}_{40}\times\underbrace{200}_{2\times100}\times\underbrace{160}_{4\times40}}$$

$$= 4\times100\times40\sqrt{2}$$

$$= 16000\sqrt{2}$$

$$= 16000\times1.414$$

$$= 22624\ m^2$$

$$= 2.26 \text{ hectare (approx.)}$$

For $\triangle ACD$, let $a = 240m$, $b = 320m$, $c = 400m$

$$S = \frac{a+b+c}{2} = \frac{240+320+400}{2} = \frac{960}{2} = 480m$$

Area ($\triangle ACD$) $= \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{480(480-240)(480-320)(480-400)}$$

$$= \sqrt{\underbrace{480}_{2\times240}\times\underbrace{240}_{240}\times\underbrace{160}_{2\times80}\times\underbrace{80}_{80}}$$

$$= 2\times240\times80 = 38400\ m^2$$

$$= 3.84 \text{ hectare}$$

Since CE is the median of ΔACD,

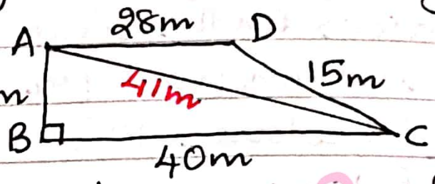
$$\text{area}(\Delta ACE) = \text{area}(\Delta CED) = \frac{1}{2}\text{area}(ACD)$$
$$= \frac{1}{2} \times 3.84$$
$$= 1.92 \text{ hectares}$$

Hence area used for growing wheat = 2.26 hectares

area used for growing potatoes = area used for growing onions = 1.92 hectares

Example: 5

Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA; while the other through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If AB = 9m, BC = 40m, CD = 15m, DA = 28m and $\angle B = 90°$, which group cleaned more area and by how much? Find the total area cleaned by the students (neglecting the width of the lanes)



Solution:-

Using Heron's formula,

for ΔADC,

let $a = 28m$, $b = 15m$, $c = 41m$

$$S = \frac{a+b+c}{2} = \frac{28+15+41}{2} = \frac{84}{2} = 42m$$

Using Pythagoras Theorem in rt. ΔABC,

$$AC^2 = AB^2 + BC^2$$
$$= 9^2 + 40^2$$
$$= 81 + 1600$$
$$= 1681$$
$$\therefore AC = \sqrt{1681} = 41m$$

$$\text{area}(\Delta ADC) = \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \sqrt{42(42-28)(42-15)(42-41)}$$
$$= \sqrt{\underset{3\times14}{42} \times \underset{14}{14} \times \underset{3\times9}{27} \times 1} = 3 \times 3 \times 14$$
$$= 126m^2$$

$$\text{area}(\Delta ABC) = \frac{1}{2} \times AB \times BC = \frac{1}{\cancel{2}_1} \times 9 \times \cancel{40}^{20} = 180m^2$$

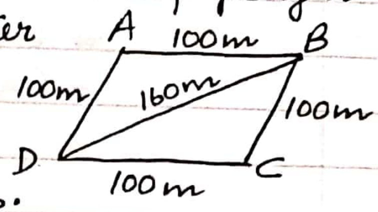
Hence group 1 students cleaned more area by

$$180 - 126 = 54m^2$$

Total area cleaned $= 180 + 126$
$$= 306m^2$$

Example 6:-

Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts.



If the perimeter of the land is 400m and one of the diagonals is 160m, how much area each of them will get for their crops?

Solution:-

Using Heron's formula, let $a = 100m$, $b = 100m$, $C = 160m$

in $\triangle ABD$, area $(\triangle ABD) = \sqrt{S(S-a)(S-b)(S-c)}$

$S = \frac{a+b+c}{2} = \frac{100+100+160}{2} = \frac{360}{2} = 180m$

$= \sqrt{180(180-100)(180-100)(180-160)}$

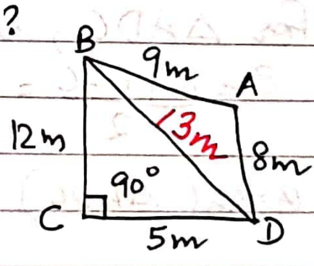
$= \sqrt{\underset{20\times 9}{180} \times 80 \times 80 \times 20}$

$= 20 \times 3 \times 80 = 4800\ m^2$

Since diagonal BD divides rhombus ABCD into two $\triangle$s of equal areas, each of them will get an area of $4800m^2$.

Exercise 12.2

1) A park, in the shape of a quadrilateral ABCD, has $\angle C = 90°$, $AB = 9m$, $BC = 12m$, $CD = 5m$ and $AD = 8m$. How much area does it occupy?



Solution:-

Area $(\triangle BCD) = \frac{1}{2} \times BC \times CD$

$= \frac{1}{\cancel{2}_1} \times \cancel{12}^{6} \times 5 = 30m^2$

Using Pythagoras Theorem in rt. $\triangle BCD$,

$BD^2 = BC^2 + CD^2 = 12^2 + 5^2 = 144 + 25 = 169$

$BD = \sqrt{169} = 13m$

Using Heron's formula in $\triangle ABD$, let $a = 9m$, $b = 8m$, $C = 13m$.

$S = \frac{a+b+c}{2} = \frac{9+8+13}{2} = \frac{30}{2} = 15m$

Area $(\triangle ABD) = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{15(15-9)(15-8)(15-13)}$

$= \sqrt{15 \times 6 \times 7 \times 2} = 3 \times 2 \times \sqrt{35}$

$\quad\; \underset{3\times5}{\wedge} \quad \underset{2\times3}{\wedge}$

$= 6\sqrt{35}$

$= 6 \times 5.91 = 35.46\ m^2$

$$\begin{array}{r|l} & 5.91 \\ \hline 5 & \overline{35}.\overline{00}\,\overline{00} \\ & 25 \\ \hline 109 & 1000 \\ & 981 \\ \hline 1181 & 1900 \\ & 1181 \\ \hline \end{array}$$

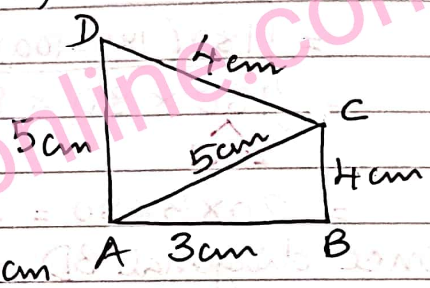
$\therefore$ area of quadrilateral ABCD

$=$ area$(\triangle BCD)$ + area$(\triangle ABD)$

$= 30 + 35.46 = \underline{\underline{65.46 m^2}}$

2) Find the area of a quadrilateral ABCD in which AB = 3cm, BC = 4cm, CD = 4cm, DA = 5cm and AC = 5cm.

Solution:-



Using Heron's formula,

in $\triangle ABC$, let $a = 3cm$, $b = 4cm$ and $c = 5cm$.

$S = \frac{a+b+c}{2} = \frac{3+4+5}{2} = \frac{12}{2} = 6cm$

area$(\triangle ABC) = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)}$

$= \sqrt{6 \times 3 \times 2 \times 1} = 3 \times 2 = 6 cm^2$

$\quad \underset{3\times2}{\wedge}$

For $\triangle ADC$, let $a = 5cm$, $b = 4cm$, $c = 5cm$

$S = \frac{a+b+c}{2} = \frac{5+4+5}{2} = \frac{14}{2} = 7cm$

area$(\triangle ADC) = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{7(7-5)(7-4)(7-5)}$

$= \sqrt{7 \times 2 \times 3 \times 2} = 2\sqrt{21}$

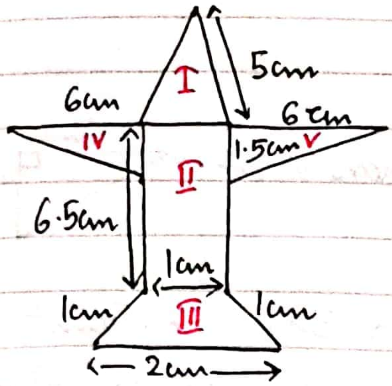
$$\begin{array}{r|l} & 4.58 \\ \hline 4 & \overline{21}.\overline{00}\,\overline{00} \\ & 16 \\ \hline 85 & 500 \\ & 425 \\ \hline 908 & 7500 \\ & 7264 \\ \hline \end{array}$$

$= 2 \times 4.58 = 9.16 cm^2$

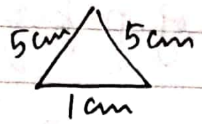
$\therefore$ area of quadrilateral ABCD

$=$ area$(\triangle ABC)$ + area$(\triangle ADC)$

$= 6 + 9.16 = \underline{\underline{15.16 cm^2}}$

3) 

Radha made a picture of an aeroplane with coloured paper. Find the total area of the paper used.

Solution:- * Area of region I :- 

let $a = 5cm, b = 5cm, C = 1cm$.

$S = \frac{a+b+c}{2} = \frac{5+5+1}{2} = \frac{11}{2}$

Using Heron's formula, area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$

$= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}$

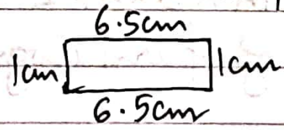
$= \frac{3\sqrt{11}}{4}$

$= \frac{3}{4} \times 3.31$

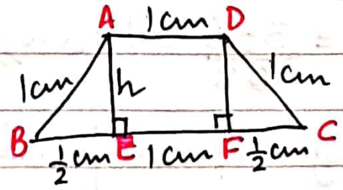
$= \frac{9.93}{4}$

$= 2.4825 cm^2$ //

$$\begin{array}{r|l} & 3.31 \\ \hline 3 & \overline{11}.\overline{00}\,\overline{00} \\ & 9 \\ \hline 63 & 200 \\ & 189 \\ \hline 661 & 1100 \\ & 661 \\ \hline \end{array}$$

* Area of region II :- 

area of rectangle $= l \times b = 6.5 \times 1 = 6.5 cm^2$ //

* Area of region III :- 

In rt. ΔABE,

$AE^2 = AB^2 - BE^2$

$= 1^2 - \left(\frac{1}{2}\right)^2$

$= 1 - \frac{1}{4} = \frac{3}{4}$

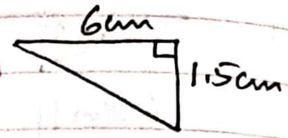
$AE = \frac{\sqrt{3}}{2} cm$

Area of trapezium $ABCD = \frac{1}{2}(AD + BC) \times AE$

$= \frac{1}{2}(1+2) \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4} = \frac{3 \times 1.732}{4}$

$$= \frac{5.196}{4} = 1.299\,cm^2$$

* Area of region IV = area of region V :–



area of right $\Delta = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 6 \times 1.5 = 3 \times 1.5 = 4.5\,cm^2$$

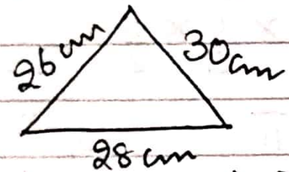
$\therefore$ Total area of paper used = area of regions I + II + III + IV + V

$$= 2.4825 + 6.5 + 1.299 + 4.5 + 4.5$$

$$= 19.2815 = 19.28\,cm^2 \text{ (approximately)}$$

4) A triangle and a parallelogram have the same base and the same area. If the sides of the Δ are 26cm, 28cm and 30cm and the parallelogram stands on the base 28cm, find the height of the parallelogram.

Solution:-



Let $a = 26cm$, $b = 30cm$, $c = 28cm$

$$S = \frac{a+b+c}{2} = \frac{26+30+28}{2} = \frac{84}{2} = 42cm$$

Using Heron's formula, area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

$$= \sqrt{\underset{14\times3}{42} \times \underset{16}{16} \times \underset{14}{14} \times \underset{3\times4}{12}}$$

$$= 14 \times 3 \times 4 \times 2 = 336\,cm^2$$

area of Δ = area of parallelogram (given)

$\Rightarrow 336 = b \times h$

$\Rightarrow 28 \times h = 336$

$$h = \frac{336}{28} = 12cm$$

Hence height of the parallelogram = 12cm

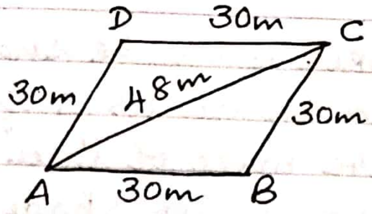
5) A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30m and its longer diagonal is 48m, how much area of grass field will each cow be getting?

Solution:-

For ΔABC, $a = 30m$, $b = 30m$, $c = 48m$

$s = \frac{a+b+c}{2} = \frac{30+30+48}{2}$

$= \frac{108}{2} = 54m$



Using Heron's formula, area($\triangle ABC$) $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-30)(54-30)(54-48)}$

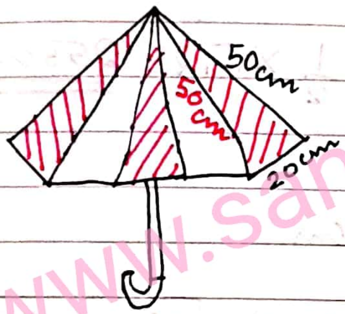
$= \sqrt{\underset{6\times 9}{\hat{54}} \times 24 \times 24 \times 6}$

$= 6 \times 3 \times 24 = 432 m^2$

Since diagonal AC divides rhombus ABCD into two $\triangle$s of equal area, area ($\triangle ABC$) = area($\triangle ADC$) $= 432m^2$

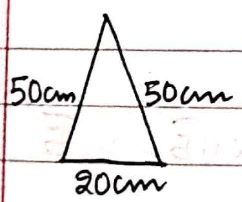
$\therefore$ Area of rhombus ABCD $= 2 \times 432 = 864 m^2$

Hence area of grass field grazed by each cow $= \frac{864}{18} = \underline{\underline{48m^2}}$

6) 

An umbrella is made by stitching 10 triangular pieces of cloth of two different colours, each piece measuring 20cm, 50cm and 50cm. How much cloth of each colour is required for the umbrella?

Solution:-



Let $a = 50cm$, $b = 50cm$, $c = 20cm$

$s = \frac{a+b+c}{2} = \frac{50+50+20}{2} = \frac{120}{2} = 60cm$

Using Heron's formula,

area of $\triangle = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{60(60-50)(60-50)(60-20)}$

$= \sqrt{\underset{6\times 10}{\hat{60}} \times 10 \times 10 \times \underset{4\times 10}{\hat{40}}} = 2 \times 10 \times 10\sqrt{6}$

$= 200\sqrt{6}\ cm^2$

$\therefore$ Area of cloth required for each colour $= 200\sqrt{6} \times \frac{10}{2}$

$= 200\sqrt{6} \times 5 = 1000\sqrt{6} = 1000 \times \sqrt{2} \times \sqrt{3}$

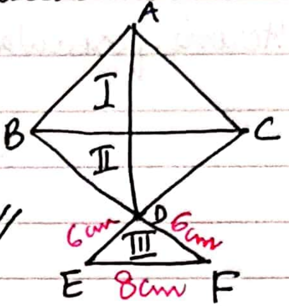
$= 1000 \times 1.414 \times 1.732$

$= 1414 \times 1.732$

$= 2449.048\ cm^2$ //

7) A kite is in the shape of a square with diagonal 32cm and an isoscles Δ of base 8cm and sides 6cm each is to be made of three different shades. How much paper of each shade has been used in it?

Solution:-



Diagonal of a Square $= \sqrt{2}\,a$

$$\sqrt{2}a = 32$$

$$a = \frac{32}{\sqrt{2}} = \frac{32\sqrt{2}}{2} = 16\sqrt{2}\,cm$$

area of Square ABDC $= a^2$

$$= 16\sqrt{2} \times 16\sqrt{2} = 512\,cm^2$$

Since diagonal BC divides the Square ABDC into two triangles of equal area,

$$area(\Delta ABC) = area(\Delta BCD) = \frac{1}{2}\, area\,(Sq.\,ABDC)$$

$$= \frac{1}{2} \times 512 = \underline{256\,cm^2}$$

For ΔDEF,

let $a = 6cm$, $b = 6cm$, $C = 8cm$

$$S = \frac{a+b+c}{2} = \frac{6+6+8}{2} = \frac{20}{2} = 10\,cm.$$

Using Heron's formula, area $(\Delta DEF) = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{10\,(10-6)(10-6)\,(10-8)}$$

$$= \sqrt{\underset{2\times 5}{10} \times 4 \times 4 \times 2} = 4 \times 2 \times \sqrt{5} = 8\sqrt{5}$$

$$= 8 \times 2.236 = \underline{17.888\,cm^2}\ (approx.)$$

Hence, area of shade I $= 256cm^2$

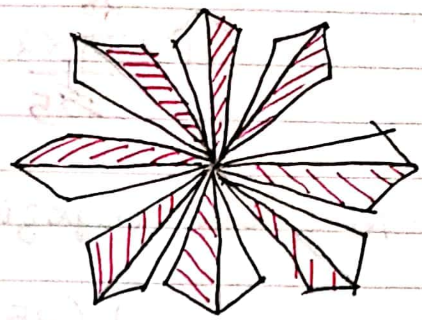
area of shade II $= 256cm^2$

area of shade III $= 17.9cm^2$

8) A floral design on a floor is made up of 16 tiles, which are triangular, the sides of the Δ being 28cm, 9cm and 35cm. Find the Cost of polishing the tiles at the rate of 50 paise per cm^2.

Solution:-

Let $a = 28cm$, $b = 9cm$, $c = 35cm$



$S = \frac{a+b+c}{2} = \frac{28+9+35}{2} = \frac{72}{2}$

$= 36cm$

area of $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{36(36-28)(36-9)(36-35)}$

$= \sqrt{36 \times \underset{4\times2}{8} \times \underset{9\times3}{27} \times 1} = 6 \times 2 \times 3\sqrt{6}$

$= 36\sqrt{6}\,cm^2$

area of 16 triangular tiles $= 36\sqrt{6} \times 16\,cm^2$

Cost of polishing $=$ area $\times$ rate

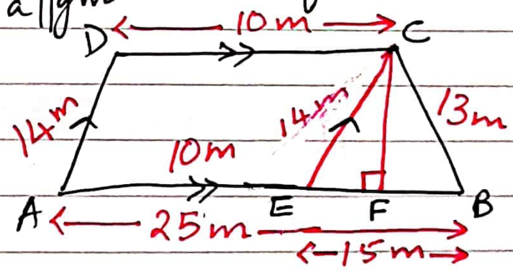
$= 36\sqrt{6} \times \cancel{16}^{8} \times \frac{\cancel{50}^{1}}{\cancel{100}_{\cancel{2}\,1}} = 36 \times 8 \times \sqrt{2} \times \sqrt{3}$

$= 288 \times 1.414 \times 1.732$

$= ₹705.32$

9) A field is in the shape of a trapezium, whose parallel sides are 25m and 10m and the non-parallel sides are 14m and 13m. Find the area of the field.

Construction: draw $CE \parallel AD$ to form AECD a $\parallel$gm. draw $CF \perp AB$.



For ΔCEB,

let $a = 14m$, $b = 13m$, $c = 25 - 10$

$= 15m$

$S = \frac{a+b+c}{2} = \frac{14+13+15}{2}$

$= \frac{42}{2} = 21m$

area $(\Delta CEB) = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{21(21-14)(21-13)(21-15)}$

$= \sqrt{\underset{7\times3}{21} \times 7 \times \underset{2\times4}{8} \times \underset{2\times3}{6}} = 7 \times 3 \times 2 \times 2$

$= 84\,m^2$

Also, area $(\Delta CEB) = \frac{1}{2} \times CF \times EB$

$\Rightarrow 84 = \frac{1}{2} \times CF \times 15$

$\therefore CF = \frac{\cancel{84}^{28} \times 2}{\cancel{15}_{5}} = \underline{\underline{\frac{56}{5} m}}$

Thus, area (trapezium ABCD) $= \frac{1}{2}(AB + CD) \times CF$

$= \frac{1}{2}(25 + 10) \times \frac{56}{5}$

$= \frac{1}{\cancel{2}_1} \times \cancel{35}^{7} \times \frac{\cancel{56}^{28}}{\cancel{5}_1} = 7 \times 28$

$= 196 m^2$

Hence, area of the field $= \underline{\underline{196 m^2}}$