IX Revision worksheet – Number Systems

1) If $x = \frac{5-\sqrt{21}}{2}$, then prove that

$$\left(x^3+\frac{1}{x^3}\right) - 5\left(x^2+\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right) = 0$$

2) Given $\sqrt{2} = 1.4142$ and $\sqrt{6} = 2.4495$. Find the value of $\frac{1}{\sqrt{3}-\sqrt{2}-1}$ correct to three places of decimal.

3) If $2^a = 3^b = 6^c$, then find the relation between a, b and c.

4) Locate $\sqrt{10}$ on the number line.

5) Prove that $\left(\frac{x^l}{x^m}\right)^{\frac{1}{lm}} \times \left(\frac{x^m}{x^n}\right)^{\frac{1}{mn}} \times \left(\frac{x^n}{x^l}\right)^{\frac{1}{nl}} = 1$

6) Find the value of $\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}}$

7) Simplify: $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6 \div \left(\frac{3^4 \times 5^2}{2^6}\right)$

8) Evaluate : $4 \times (81)^{-\frac{1}{2}} \times \left[81^{\frac{1}{2}} + 81^{\frac{3}{2}}\right]$

9) Simplify: $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

10) Express $2.543434 3\ldots$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

11) Locate $\sqrt{3}$ on the number line.

12) Let x and y be rational and irrational number respectively. Then $(x+y)$ is necessarily
(a) a whole number (b) a rational number
(c) an irrational number (d) a natural number

13) The value of x, if $\sqrt[3]{4x-7} = 5$ is

(a) 23 (b) 39 (c) 33 (d) 34

14) Write two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.

15) Simplify: $64^{-\frac{1}{3}} \cdot 64^{\frac{1}{3}} - 343^{\frac{2}{3}}$

16) Insert two rational numbers between $\frac{2}{3}$ and $\frac{5}{3}$

17) On simplifying $\left(\sqrt[3]{x^2}\right)^{\frac{3}{2}}$, we get

(a) $\sqrt{x}$ (b) x (c) $x^{\frac{3}{2}}$ (d) 1

18) On simplifying $(5+\sqrt{2})(3+\sqrt{5})$, we get

(a) $15+5\sqrt{5}+3\sqrt{2}+\sqrt{10}$

(b) $15+9\sqrt{10}$

(c) 105

(d) $15+5\sqrt{5}+2\sqrt{3}+\sqrt{10}$

19) Give examples of two irrational numbers, the product of which is

(i) a rational number (ii) an irrational number.

20) The number 1.010010001... is

(a) a natural number (b) a whole number

(c) a rational number (d) an irrational number.

IX

Revision Worksheet – Number Systems (Solutions)

1) $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$a^2+b^2 = (a+b)^2 - 2ab$

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$

Given, $x = \frac{5-\sqrt{21}}{2}$

Then, $\frac{1}{x} = \frac{2}{5-\sqrt{21}} = \frac{2(5+\sqrt{21})}{(5-\sqrt{21})(5+\sqrt{21})}$

$= \frac{2(5+\sqrt{21})}{(5)^2-(\sqrt{21})^2} = \frac{2(5+\sqrt{21})}{25-21} = \frac{2(5+\sqrt{21})}{4}$

$= \frac{5+\sqrt{21}}{2}$

$\therefore x^3+\frac{1}{x^3} = \left(\frac{5-\sqrt{21}}{2}+\frac{5+\sqrt{21}}{2}\right)^3 - 3\left(\frac{5-\sqrt{21}}{2}+\frac{5+\sqrt{21}}{2}\right)$

$= \left(\frac{10-\sqrt{21}+\sqrt{21}}{2}\right)^3 - 3\left(\frac{10-\sqrt{21}+\sqrt{21}}{2}\right)$

$= \left(\frac{10}{2}\right)^3 - 3\left(\frac{10}{2}\right) = 5^3 - 3\times5 = 125-15$

$= \underline{\underline{110}}$ | $x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2x\times\frac{1}{x}$

$\therefore x^2+\frac{1}{x^2} = \left(\frac{5-\sqrt{21}}{2}+\frac{5+\sqrt{21}}{2}\right)^2 - 2\left(\frac{5-\sqrt{21}}{2}\right)\left(\frac{5+\sqrt{21}}{2}\right)$

$= \left(\frac{5-\sqrt{21}+5+\sqrt{21}}{2}\right)^2 - 2\left(\frac{25-21}{4}\right)$

$= \left(\frac{10}{2}\right)^2 - 2\times\frac{4}{4} = 5^2-2 = 25-2 = \underline{\underline{23}}$

$x+\frac{1}{x} = \frac{5-\sqrt{21}}{2}+\frac{5+\sqrt{21}}{2} = \frac{10}{2} = \underline{\underline{5}}$

$\therefore \left(x^3+\frac{1}{x^3}\right) - 5\left(x^2+\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right) = 110-5\times23+5$

$= 110-115+5$

$= -5+5 = 0$

Hence proved

2) Given, $\sqrt{2} = 1.4142$
$\sqrt{6} = 2.4495$

$$\frac{1}{(\sqrt{3}-\sqrt{2})-1} = \frac{(\sqrt{3}-\sqrt{2})+1}{[(\sqrt{3}-\sqrt{2})-1][(\sqrt{3}-\sqrt{2})+1]}$$

$$= \frac{\sqrt{3}-\sqrt{2}+1}{(\sqrt{3}-\sqrt{2})^2-1^2} = \frac{\sqrt{3}-\sqrt{2}+1}{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}\times\sqrt{2}-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}+1}{3+2-2\sqrt{6}-1} = \frac{\sqrt{3}-\sqrt{2}+1}{4-2\sqrt{6}}$$

$$= \frac{\sqrt{3}-\sqrt{2}+1}{2(2-\sqrt{6})}$$

$$= \frac{(\sqrt{3}-\sqrt{2}+1)(2+\sqrt{6})}{2(2-\sqrt{6})(2+\sqrt{6})}$$

$$= \frac{2\sqrt{3}+\sqrt{18}-2\sqrt{2}-\sqrt{12}+2+\sqrt{6}}{2(4-6)}$$

$$= \frac{2\sqrt{3}+3\sqrt{2}-2\sqrt{2}-2\sqrt{3}+2+\sqrt{6}}{-4}$$

$$= \frac{\sqrt{2}+2+\sqrt{6}}{-4}$$

$$= \frac{1.4142+2+2.4495}{-4}$$

$$= \frac{-5.8637}{4} = -1.465925$$

$$= -1.466 \text{ (approx.)}$$

$2 | 18$
$3 | 9$
3

$2 | 12$
$2 | 6$
3

3) $2^a = 6^c$

$\Rightarrow 2 = 6^{\frac{c}{a}} \longrightarrow (1)$

$3^b = 6^c$

$\Rightarrow 3 = 6^{\frac{c}{b}} \longrightarrow (2)$

$\therefore 6 = 2 \times 3 = 6^{\frac{c}{a}} \times 6^{\frac{c}{b}}$

$\Rightarrow 6^1 = 6^{\frac{c}{a}+\frac{c}{b}}$

$$\therefore 1 = \frac{c}{a} + \frac{c}{b}$$

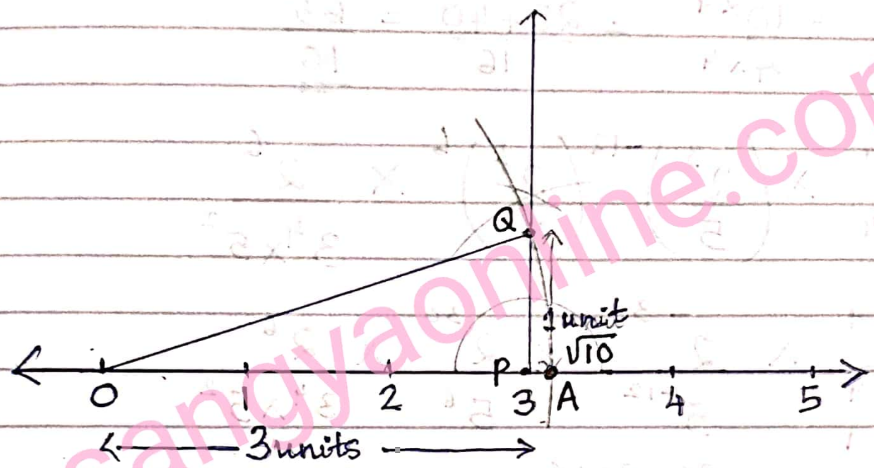
$$\Rightarrow 1 = c\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\Rightarrow 1 = c\left(\frac{b+a}{ab}\right)$$

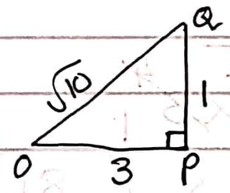
$$\Rightarrow \frac{ab}{a+b} = c$$

$$\therefore c = \frac{ab}{a+b}$$

4)



Using Pythagoras Theorem,



$$OQ^2 = OP^2 + OQ^2$$
$$= 3^2 + 1^2$$
$$= 9 + 1 = 10$$
$$\therefore OQ = \sqrt{10} \text{ units}$$

5) LHS,
$$\frac{x^{l \times \frac{1}{lm}}}{x^{m \times \frac{1}{lm}}} \times \frac{x^{m \times \frac{1}{mn}}}{x^{n \times \frac{1}{mn}}} \times \frac{x^{n \times \frac{1}{nl}}}{x^{l \times \frac{1}{nl}}}$$

$$= \frac{x^{\frac{1}{m}}}{x^{\frac{1}{l}}} \times \frac{x^{\frac{1}{n}}}{x^{\frac{1}{m}}} \times \frac{x^{\frac{1}{l}}}{x^{\frac{1}{n}}} = 1, \text{ RHS}$$

6) $\left(\frac{4^3}{5^3}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + \frac{5}{4}$

$= \frac{4^{3\times-\frac{2}{3}}}{5^{3\times-\frac{2}{3}}} + \frac{1}{\frac{4^{4\times\frac{1}{4}}}{5^{4\times\frac{1}{4}}}} + \frac{5}{4}$

$= \frac{4^{-2}}{5^{-2}} + \frac{5}{4} + \frac{5}{4} = \frac{5^2}{4^2} + \frac{5}{4} + \frac{5}{4}$

$= \frac{25}{16} + \frac{10\times4}{4\times4} = \frac{25+40}{16} = \frac{65}{16}$

7) $\frac{3^4}{5^4} \times \left(\frac{2^3}{5}\right)^{-12} \left(\frac{2^5}{5}\right)^{6} \times \frac{2^6}{3^4\times5^2}$

$= \frac{3^4}{5^4} \times \frac{2^{-36}}{5^{-12}} \times \frac{2^{30}}{5^6} \times \frac{2^6}{3^4\times5^2}$

$= \frac{3^4 \times 2^{-36+30+6}}{5^{4-12+6+2} \times 3^4} = \frac{2^0}{5^0} = \frac{1}{1} = 1$

8) $4 \times (81)^{-\frac{1}{2}} \times \left(81^{\frac{1}{2}} + 81^{\frac{3}{2}}\right)$

$= 4 \times 9^{2\times-\frac{1}{2}} \times \left(9^{2\times\frac{1}{2}} + 9^{2\times\frac{3}{2}}\right)$

$= 4\times9^{-1}\,(9+9^3)$

$= \frac{4}{9}\times9(1+9^2) = 4\times82 = 328$

9) $\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{18}-\sqrt{12}}{3-2} = \sqrt{18}-\sqrt{12}$

$\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} = \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})} = \frac{3(\sqrt{12}-\sqrt{6})}{6-3} = \frac{3(\sqrt{12}-\sqrt{6})}{3}$

$= \sqrt{12}-\sqrt{6}$

$$\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} = \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} = \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2} = \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{4}$$

$$= \sqrt{18}-\sqrt{6}$$

$$\therefore \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} = \sqrt{18}-\sqrt{12}+\sqrt{12}-\sqrt{6}-\sqrt{18}+\sqrt{6}$$

$$= 0$$

10) Let $2.5434343\ldots = x$

$10x = 25.434343\ldots \rightarrow (1)$

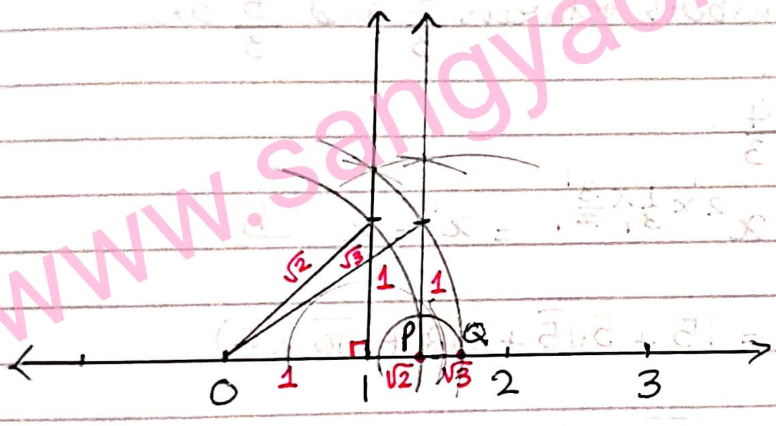
$1000x = 2543.434343\ldots \rightarrow (2)$

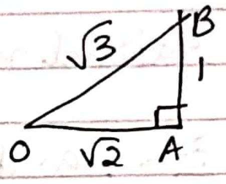
(2)−(1), $1000x - 10x = 2543 - 25$

$\Rightarrow 990x = 2518$

$x = \frac{2518}{990} = \frac{1259}{495}$, which is in the $\frac{p}{q}$ form.

11)





Using Pythagoras Theorem,

$OB^2 = OA^2 + AB^2$

$= (\sqrt{2})^2 + 1^2$

$= 2+1 = 3$

$\therefore OB = \sqrt{3}$ units

Thus Q represents $\sqrt{3}$ on the number line.

12) an irrational number (c)

eg:- $2+\sqrt{3}$ is an irrational number.

13)

$\sqrt[3]{4x-7} = 5$

Taking cubes on both sides,

$4x - 7 = 5^3$

$\Rightarrow 4x = 125 + 7 = 132$

$\therefore x = \frac{132}{4} = 33$

14) $\sqrt{2} = 1.414$
$\sqrt{3} = 1.732$
Two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$ are
1.5050050005...
1.6262262226....

15)
$64^{-\frac{1}{3}} . 64^{\frac{1}{3}} - 343^{\frac{2}{3}}$

$= 4^{3 \times -\frac{1}{3}} \times 4^{3 \times \frac{1}{3}} - 7^{3 \times \frac{2}{3}}$

$= 4^{-1} \times 4 - 7^2$

$= \frac{4}{4} - 49$

$= 1 - 49 = \underline{-48}$

16) Two rational numbers between $\frac{2}{3}$ and $\frac{5}{3}$ are

$\frac{3}{3} = 1$ and $\frac{4}{3}$

17) $\left(\sqrt[3]{x^2}\right)^{\frac{3}{2}} = x^{2 \times \frac{1}{3} \times \frac{3}{2}} = x^1 = x$ (b)

18) $(5+\sqrt{2})(3+\sqrt{5}) = 15 + 5\sqrt{5} + 3\sqrt{2} + \sqrt{10}$ (a)

19) (i) $\sqrt{27} \times \sqrt{3} = \sqrt{81} = 9$, which is a rational number.
(ii) $\sqrt{2} \times \sqrt{5} = \sqrt{10}$, which is an irrational number.

20) 1.010010001... is an irrational number.
ie, it has non-terminating non-repeating decimal expansion.