X HW-10

1) If the ratio of the sum of the first n terms of two A.Ps is $(7n+1):(4n+27)$, then find the ratio of their 9^{th} terms.

2) The Sum of four consecutive numbers in an A.P is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7:15. Find the no.s

3) If the ratio of the 11^{th} term of an A.P to its 18^{th} term is 2:3, find the ratio of the sum of the first five terms to the sum of its first 10 terms.

4) If the p^{th} term of an A.P is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$,

Prove that the Sum of first pq terms of the A.P is $\left(\frac{pq+1}{2}\right)$

5) Find the sum of first 24 terms of an A.P whose n^{th} term is given by $a_n = 3+2n$.

6) If $1+4+7+10+\cdots+x = 287$, find the value of x.

7) In a garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row. How many rows are there of rose plants? Also, find the total no. of rose plants.

8) If the sum of first n terms of an A.P is given by $S_n = 3n^2+4n$. Determine the A.P and the n^{th} term.

9) Find the middle term of the sequence formed by numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately.

10) Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also, find the sum of all numbers on both sides of the middle term separately.

11) The Sum of the 3^{rd} and 7^{th} terms of an A.P is 6 and their product is 8. Find the sum of first 20 terms of A.P.

MCQs

12) The common difference of the A.P: $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is
(a) p (b) $-p$ (c) -1 (d) 1

13) The 4^{th} term from the end of the A.P: $-11, -8, -5, \ldots 49$ is
(a) 37 (b) 40 (c) 43 (d) 58

14) If 7 times the 7^{th} term of an A.P is equal to 11 times its 11^{th} term, then its 18^{th} term will be (a) 7 (b) 11 (c) 18 (d) 0.

15) Two A.Ps have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4^{th} terms is: (a) -1 (b) -8 (c) 7 (d) -9

16) What is the c.d of an A.P in which $a_{18} - a_{14} = 32$?
(a) 8 (b) -8 (c) -4 (d) 4

17) If the c.d of an A.P is 5, then what is $a_{18} - a_{13}$?
(a) 5 (b) 20 (c) 25 (d) 30

18) Which term of the A.P: 21, 42, 63, 84, ... is 210?
(a) 9^{th} (b) 10^{th} (c) 11^{th} (d) 12^{th}

19) If the 2^{nd} term of an A.P is 13 and the 5^{th} term is 25, what is its 7^{th} term? (a) 30 (b) 33 (c) 37 (d) 38

20) The 21^{st} term of an A.P, whose first two terms are -3 and 4 is
(a) 17 (b) 137 (c) 143 (d) -143

21) The first four terms of an A.P, whose first term is -2 and the c.d is -2 are: (a) $-2, 0, 2, 4$ (b) $-2, 4, -8, 16$ (c) $-2, -4, -6, -8$.

22) The 11^{th} term of the A.P: $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$ is (a) -20 (b) 20 (c) -30

23) The list of no.s $-10, -6, -2, 2, \ldots$ is
(a) an A.P with $d = -16$ (b) an A.P with $d = 4$ (c) an A.P with $d = -4$ (d) not an A.P

24) In an A.P, $a = 3.5, d = 0, n = 101$, then $a_n =$
(a) 0 (b) 103.5 (c) 3.5 (d) 104.5

25) 11^{th} term of the A.P: $-3, -\frac{1}{2}, 2, \ldots$ is (a) 28 (b) 22 (c) -38 (d) $-48\frac{1}{2}$

26) In an A.P, if $d = -4, n = 7, a_n = 4$, then $a =$ (a) 6 (b) 7 (c) 20 (d) 28

X Homework-10 (Arithmetic Progression)

1) Let a, A and d, D be the first terms and the common differences of the given A.P

ATQ, $\dfrac{\cancel{\frac{n}{2}}[2a+(n-1)d]}{\cancel{\frac{n}{2}}[2A+(n-1)D]} = \dfrac{7n+1}{4n+27}$

$\div(2)$, $\dfrac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \dfrac{7n+1}{4n+27}$

Now, put $\frac{n-1}{2} = 9-1$ $[\because a_n = a+(n-1)d]$

$\Rightarrow n-1 = 16$

$\Rightarrow n = 17$

On substituting $n = 17$, the ratio of 9^{th} terms

$= \dfrac{7\times17+1}{4\times17+27} = \dfrac{\cancel{120}^{24}}{\cancel{95}_{19}} = \dfrac{24}{19}$

Hence the required ratio = 24:19

2) Let the four consecutive numbers of the given A.P be $(a-3d), (a-d), (a+d)$ and $(a+3d)$.

ATQ, $a-\cancel{3d}+a-\cancel{d}+a+\cancel{d}+a+\cancel{3d} = 32$

$4a = 32$

$a = \frac{32}{4} = 8 \longrightarrow (1)$

Also, $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$

$\Rightarrow \dfrac{a^2-9d^2}{a^2-d^2} = \dfrac{7}{15}$

$\Rightarrow 15a^2-135d^2 = 7a^2-7d^2$

$\Rightarrow 8a^2 = 128d^2$

$\Rightarrow a^2 = 16d^2$

$\Rightarrow 64 = 16d^2$ [from eq:(1)]

$\therefore d^2 = \frac{64}{16} = 4$

$d = \sqrt{4} = \pm 2$

When $a = 8, d = +2$, the numbers are $a-3d = 8-6 = 2$,
$a-d = 8-2 = 6$,
$a+d = 8+2 = 10$,
$a+3d = 8+6 = 14$,

And when $a = 8, d = -2$, then the numbers are $a-3d = 8+6 = 14$,
$a-d = 8+2 = 10$,
$a+d = 8-2 = 6$,
$a+3d = 8-6 = 2$,

3) Let a and d be the first term and the common difference of the given A.P

$a_n = a+(n-1)d$; $S_n = \frac{n}{2}[2a+(n-1)d]$

ATQ, $\frac{a_{11}}{a_{18}} = \frac{2}{3}$

$\Rightarrow \frac{a+10d}{a+17d} = \frac{2}{3}$

$\Rightarrow 3a+30d = 2a+34d$

$\Rightarrow a = 4d$

Then, $\frac{S_5}{S_{10}} = \frac{\frac{5}{2}[2a+4\times d]}{\frac{10}{2}[2a+9\times d]} = \frac{\cancel{5}^{1}}{\cancel{10}_{2}} \frac{(2a+4d)}{(2a+9d)}$

$= \frac{8d+4d}{2(8d+9d)} = \frac{\cancel{12d}^{6}}{\cancel{2}\times 17\cancel{d}} = \frac{6}{17}$

$\therefore$ The required ratio is $6:17$

4) Let a and d be the first term and the common difference of the given A.P

$a_n = a+(n-1)d$; $S_n = \frac{n}{2}[2a+(n-1)d]$

ATQ, $a_p = \frac{1}{q} \Rightarrow a+(p-1)d = \frac{1}{q} \longrightarrow (1)$

$a_q = \frac{1}{p} \Rightarrow a+(q-1)d = \frac{1}{p} \longrightarrow (2)$

(1)−(2), $d(p-1)-d(q-1)=\frac{1}{q}-\frac{1}{p}$

$$\Rightarrow d(p-\cancel{1}-q+\cancel{1})=\frac{p-q}{pq}$$

$$\Rightarrow d(\cancel{p-q})=\frac{\cancel{p-q}}{pq}$$

$$\therefore d=\frac{1}{pq}$$

From eq:(1), $a+(p-1)\times\frac{1}{pq}=\frac{1}{q}$

$$\Rightarrow a+\frac{\cancel{p}}{\cancel{p}q}-\frac{1}{pq}=\frac{1}{q}$$

$$\Rightarrow a+\cancel{\frac{1}{q}}-\frac{1}{pq}=\cancel{\frac{1}{q}}$$

$$\therefore a=\frac{1}{pq}$$

$$\therefore S_{pq}=\frac{pq}{2}[2a+(pq-1)d]$$

$$=\frac{pq}{2}\left[\frac{2}{pq}+\frac{1}{pq}(pq-1)\right]$$

$$=\frac{pq}{2}\left[\frac{2}{pq}+\frac{\cancel{pq}}{\cancel{pq}}-\frac{1}{pq}\right]$$

$$=\frac{pq}{2}\left[\frac{1}{pq}+1\right]$$

$$=\frac{\cancel{pq}}{2}\left(\frac{1+pq}{\cancel{pq}}\right)=\frac{pq+1}{2}$$

5) $a_n=3+2n$

first term, $a_1=3+2\times1=3+2=5$

Second term, $a_2=3+2\times2=3+4=7$

$\therefore$ Common difference, $d=a_2-a_1=7-5=2$

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_{24}=\frac{\overset{12}{\cancel{24}}}{\cancel{2}}[2\times5+23\times2]$$

$= 12[10+46]$
$= 12 \times 56 = \underline{672}$

6) Let a and d be the first term and the common difference

$a = 1$
$d = 4-1 = 3$
$a_n = x$
$S_n = 287$

$* \; a_n = a+(n-1)d$

$\Rightarrow x = 1+(n-1)3$

$\Rightarrow \frac{x-1}{3} = n-1$

$\Rightarrow n = \frac{x-1}{3}+1 = \frac{x-1+3}{3} = \frac{x+2}{3}$ //

$* \; S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 287 = \frac{x+2}{6}\left[2+\left(\frac{x-1}{\cancel{3}_1}\right)\times\cancel{3}^1\right]$

$\Rightarrow 287 \times 6 = (x+2)(x+1)$
$\Rightarrow x^2+3x+2 = 1722$
$\Rightarrow x^2+3x-1720 = 0$
$\Rightarrow (x-40)(x+43) = 0$
$\therefore x = 40, -43$

S	P
3	-1720

$-40, 43$

$$\begin{array}{r|l} 2 & 1720 \\ 2 & 860 \\ 2 & 430 \\ 5 & 215 \\ & 43 \end{array}$$

x cannot be -ve,
$\therefore$ Required value of $x = 40$ //

7) The number of rose plants in $1^{st}, 2^{nd}, 3^{rd}$ ... rows are 23, 21, 19, ... 5 forms an A.P with first term, $a = 23$ and common difference, $d = 21-23 = -2$.

$* \; a_n = 5$
$\Rightarrow a+(n-1)d = 5$
$\Rightarrow 23 - 2(n-1) = 5$
$\Rightarrow -2(n-1) = -18$
$\Rightarrow n-1 = 9$
$\therefore n = 10$ /

$* \; S_n = \frac{n}{2}[a + a_n]$

$\Rightarrow S_{10} = \frac{10}{2}[23+5]$

$= 5 \times 28 = \underline{140}$

Hence the total number of rose plants $= 140$
and no. of rows $= 10$

8) $S_n = 3n^2 + 4n$

Then, $a_1 = S_1 = 3 \times 1^2 + 4 \times 1 = 3 + 4 = 7$

Also, $a_1 + a_2 = S_2 = 3 \times 2^2 + 4 \times 2 = 12 + 8 = 20$

$\Rightarrow a_2 = S_2 - S_1 = 20 - 7 = 13$

$\therefore d = a_2 - a_1 = 13 - 7 = 6$

Hence the required A.P is $a_1, a_2, a_3, a_4, \ldots$

$\Rightarrow a, a+d, a+2d, a+3d, \ldots$

$\Rightarrow 7, 13, 19, 25, \ldots$

n^{th} term, $a_n = a + (n-1)d$

$= 7 + (n-1) \times 6$

$= 7 + 6n - 6$

$\underline{\underline{a_n = 6n + 1}}$

9) The numbers $10, 13, 16, \ldots 94$ forms an A.P with first term, $a = 10$ and common difference, $d = 13 - 10 = 3$, $a_n = 94$.

$* \; a_n = a + (n-1)d$

$94 = 10 + (n-1) \times 3$

$\frac{94 - 10}{3} = n - 1$

$n - 1 = \frac{84}{3} = 28$

$\therefore n = 29$

Middle term $= \left(\frac{n+1}{2}\right)^{th}$ term $= 15^{th}$ term $= a_{15}$

$= a + 14d = 10 + 14 \times 3$

$= 10 + 42 = \underline{\underline{52}}$

$* \ S_n = \frac{n}{2}[2a+(n-1)d]$

Sum of first 14 terms $= S_{14} = \frac{14}{2}[2\times10+13\times3]$

$= 7[20+39]$

$= 7\times59 = \underline{\underline{413}}$

Sum of last 14 terms $= S_{29} - S_{15}$

$= \frac{29}{2}[20+28\times3] - \frac{15}{2}[20+14\times3]$

$= \frac{29}{\cancel{2}_1}\times\cancel{104}^{52} - \frac{15}{\cancel{2}_1}\times\cancel{62}^{31}$

$= 1508 - 465$

$= \underline{\underline{1043}}$

10) The numbers 103, 110, 117, ... 999 forms an A.P with the first term, $a = 103$ and the common difference, $d = 110-103 = 7$.

$* \ a_n = 999$

$\Rightarrow a+(n-1)d = 999$

$\Rightarrow 103+(n-1)\times7 = 999$

$\Rightarrow n-1 = \frac{999-103}{7} = 128$

$n = 129$ //

$\therefore$ Middle term $= \left(\frac{n+1}{2}\right)^{th}$ term $= 65^{th}$ term $= a_{65}$

$= a+64d = 103+64\times7$

$= 103+448$

$= \underline{551}$

$*$ Sum of first 64 terms $= S_{64} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{64}{2}[2\times103+63\times7]$

$= 32[206+441]$

$= 32\times647 = 20704$ //

$*$ Sum of last 64 terms $= S_{129} - S_{65}$

$= \frac{129}{2}[206+128\times7] - \frac{65}{2}[206+64\times7]$

$= \frac{129}{\cancel{2}_1}\times\cancel{1102}^{551} - \frac{65}{\cancel{2}_1}\times\cancel{654}^{327}$

$= 129 \times 551 - 65 \times 327$

$= 71079 - 21255$

$= \underline{49824}$

11) Let the first term and the common difference of the given A.p be a and d respectively.

$a_n = a + (n-1)d$

ATQ, $a_3 + a_7 = 6$

$\Rightarrow a + 2d + a + 6d = 6$

$\Rightarrow 2a + 8d = 6$

$(\div 2) \Rightarrow a + 4d = 3 \rightarrow (1)$

Also, $a_3 \times a_7 = 8$

$(a+2d)(a+6d) = 8$

$\Rightarrow (a+4d-2d)(a+4d+2d) = 8$

$\Rightarrow (3-2d)(3+2d) = 8$ [from eq:(1)]

$\Rightarrow 9 - 4d^2 = 8$

$\Rightarrow -4d^2 = -1$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d^2 = \frac{1}{4}$

$\therefore d = \pm \frac{1}{2}$

Case 1:- from eq:(1), when $d = \frac{1}{2}$,

$a = 3 - \frac{4}{2} = 3 - 2$

$a = 1$

Then $S_{20} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}\left[2 + 19 \times \frac{1}{2}\right] = \cancel{10}^{5} \times \frac{(4+19)}{\cancel{2}}$

$= 5 \times 23 = \underline{\underline{115}}$

$$\begin{array}{r} 551 \\ \times 129 \\ \hline 4959 \\ 1102 \\ 551 \\ \hline 71079 \end{array} \qquad \begin{array}{r} 327 \\ \times 65 \\ \hline 1635 \\ 1962 \\ \hline 21255 \end{array}$$

Case 2 :- When $d = -\frac{1}{2}$

from eq:(1), $a = 3 + \frac{4}{2} = 3 + 2 = 5$

Then $S_{20} = \frac{20}{2}\left[2 \times 5 + 19 \times -\frac{1}{2}\right]$

$= 10\left[10 - \frac{19}{2}\right] = 10 \times \frac{(20-19)}{2}$

$= 5 \times 1 = \underline{\underline{5}}$

MCQs

12) $d = a_2 - a_1 = \frac{1-p}{p} - \frac{1}{p}$

$= \frac{\cancel{1} - p - \cancel{1}}{p} = -1$ (c)

13) n^{th} end from the end of A.P $= l - (n-1)d$ | $d = -8+11 = 3$

$= 49 - 3 \times 3$

$= 49 - 9 = 40$ (b)

14) $7a_7 = 11a_{11}$

$\Rightarrow 7(a+6d) = 11(a+10d)$

$\Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow -4a - 68d = 0$

$(\div -4) \Rightarrow a + 17d = 0 \rightarrow (1)$

$\therefore a_{18} = a + 17d = 0$ (d)

15) Let $a = -1$, $A = -8$ and d be the common difference

Then $a_4 - A_4 = (a+3d) - (A+3d)$

$= a + \cancel{3d} - A - \cancel{3d}$

$= -1 + 8 = \underline{\underline{7}}$ (a)

16) $a_{18} - a_{14} = 32$

$\Rightarrow a + 17d - (a + 13d) = 32$

$\Rightarrow \cancel{a} + 17d - \cancel{a} - 13d = 32$

$\therefore 4d = 32$

$d = 8$ // (a)

17) $d = 5$

$a_{18} - a_{13} = a + 17d - (a + 12d)$

$= \cancel{a} + 17d - \cancel{a} - 12d$

$= 5d = 5 \times 5 = \underline{\underline{25}}$ (c)

18) $a = 21$

$d = 42 - 21 = 21$

$a_n = 210$

$\Rightarrow a_n = a + (n-1)d$

$\Rightarrow 210 = 21 + (n-1)21$

$\Rightarrow \frac{210 - 21}{21} = n - 1$

$\therefore n - 1 = 9$

$n = \underline{\underline{10}}^{th}$ term (b)

19) $a_2 = 13 \Rightarrow a + d = 13 \rightarrow (1)$

$a_5 = 25 \Rightarrow a + 4d = 25 \rightarrow (2)$

$(1) - (2), \; -3d = -12$

$d = 4$

From eq: (1), $a + 4 = 13$

$a = 9$

$\therefore a_7 = a + 6d = 9 + 6 \times 4 = 9 + 24 = \underline{\underline{33}}$ (b)

20) $a_1 = -3$

$a_2 = 4$

$d = a_2 - a_1 = 4 + 3 = 7$

$\therefore a_{21} = a + 20d = -3 + 20 \times 7 = -3 + 140 = \underline{\underline{137}}$ (b)

21) $a = -2$

$d = -2$

first 4 terms are $-2, -2-2 = -4, -4-2 = -6$ and $-6-2 = -8$

i.e, $-2, -4, -6, -8$ (c)

22) $a = -5$

$d = -\frac{5}{2} + 5 = \frac{-5 + 10}{2} = \frac{5}{2}$

$a_{11} = a + 10d = -5 + \cancel{10}^{5} \times \frac{5}{\cancel{2}_1} = -5 + 25 = \underline{\underline{20}}$ (b)

23) $a_2 - a_1 = -6 + 10 = 4$

$a_3 - a_2 = -2 + 6 = 4$

$a_4 - a_3 = 2 + 2 = 4$

an A.P with $d = 4$ (b)

24) $a_n = a + (n-1)d$

$= 3.5 + 100 \times 0$

$= 3.5$ (c)

25) $a = -3$

$d = -\frac{1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$

$\therefore a_{11} = a + 10d = -3 + \cancel{10}^{5} \times \frac{5}{\cancel{2}_1} = -3 + 25 = 22$ (b)

26) $a_n = a + (n-1)d$

$4 = a - 4(7-1)$

$4 = a - 4 \times 6$

$a = 4 + 24 = 28$ (d)