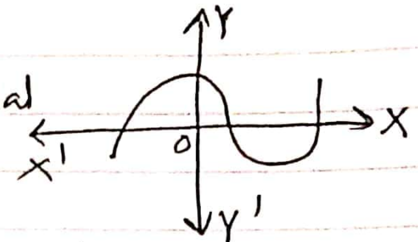
X Homework-6

1) Find the no. of zeroes of the polynomial given in the graph



2) If 1 is zero of polynomial $p(x) = ax^2 - 3(a-1)x - 1$, then find the value of a.
3) If α and β are zeroes of a polynomial such that $\alpha + \beta = 6$ and $\alpha\beta = 6$, then write a polynomial.
4) Write the polynomial, the product and sum of whose zeroes are $-\frac{9}{2}$ and $-\frac{3}{2}$ respectively.
5) Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$
6) If two zeroes of polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.
7) $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$, $g(x) = x^3 - 3x + 1$, check whether $g(x)$ is a factor of $p(x)$.
8) If $\frac{2}{3}$ and -3 are the zeroes of the polynomial $ax^2 + 7x + b$, then find values of a and b.
9) Find all zeroes of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$. If two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.
10) Find all the zeroes of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.
11) On dividing $x^3 - 3x^2 + x - 2$ by a polynomial $g(x)$, the quotient and remainder were $(x-2)$ and $-2x+4$ respectively. Find $g(x)$
12) If the polynomial $6x^4 + 8x^3 + 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b.
13) If $\frac{2}{3}$ and -3 are the zeroes of the polynomial $ax^2 + 7x + b$, find the values of a and b.
14) If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$, the remainder is $(x+a)$. Find the values of k and a.

X Homework-6 (Solutions)

1) Since the graph intersects the x-axis at 3 points, the no. of zeroes = 3

2) Since 1 is zero of $p(x)$, $p(1) = 0$

$$\Rightarrow a(1)^2 - 3(a-1)1 - 1 = 0$$
$$\Rightarrow a - 3a + 3 - 1 = 0$$
$$\Rightarrow -2a + 2 = 0$$
$$\Rightarrow -2a = -2$$
$$\therefore \underline{\underline{a = 1}}$$

3) The required polynomial is $x^2 - (\alpha+\beta)x + \alpha\beta$
$$= x^2 - 6x + 6 //$$

4) Let the zeroes be α and β.

$$\alpha\beta = -\frac{9}{2}$$

$$\alpha + \beta = -\frac{3}{2}$$

$\therefore$ The required polynomial is $x^2 - (\alpha+\beta)x + \alpha\beta$
$$= x^2 + \frac{3}{2}x - \frac{9}{2}$$

$$= \frac{1}{2}(2x^2 + 3x - 9) //$$

5) Let $p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$
$$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$$
$$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$$
$$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$$

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-8 $\quad 12 < \begin{matrix} -6 \\ -2 \end{matrix}$

$\therefore$ The zeroes of $p(x)$ are $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ and $2\sqrt{3}$.

6) Let $p(x) = x^3 - 4x^2 - 3x + 12$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of $p(x)$, then $(x - \sqrt{3})$ and $(x + \sqrt{3})$ are its factors.

Also $x^2 - 3$ is a factor of $p(x)$.

On dividing $p(x)$ by $x^2 - 3$,

$$\begin{array}{r|l} & x - 4 \\ \hline x^2 - 3 & \cancel{x^3} - 4x^2 - \cancel{3x} + 12 \\ & \underset{(-)}{\cancel{x^3}} + 0x^2 \underset{(+)}{-} \cancel{3x} \\ \hline & -4\cancel{x^2} + \cancel{12} \\ & \underset{(+)}{-}4\cancel{x^2} \underset{(-)}{+} \cancel{12} \\ \hline & \underline{0} \end{array}$$

Using division algorithm,

$p(x) = (x^2 - 3)(x - 4)$

$\therefore$ The third zero is $\underline{4}$

7) $p(x) = x^5 - 4x^3 + x^2 + 3x + 1$

$g(x) = x^3 - 3x + 1$

On dividing $p(x)$ by $g(x)$,

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3 - 3x + 1 & \cancel{x^5} - 4x^3 + \cancel{x^2} + 3x + 1 \\
 & \underset{(-)}{\cancel{x^5}} \underset{(+)}{-} 3x^3 \underset{(-)}{+} \cancel{x^2} \\
\hline
 & -\cancel{x^3} + 3\cancel{x} + 1 \\
 & \underset{(+)}{-} \cancel{x^3} \underset{(-)}{+} \cancel{3x} \underset{(+)}{-} 1 \\
\hline
 & \underline{\underline{2}} \neq 0
\end{array}
$$

Since the remainder is 2,
$g(x)$ is not a factor of $p(x)$

8) Let $p(x) = ax^2 + 7x + b$.

Since $\frac{2}{3}$ is a zero of $p(x)$, then $p\left(\frac{2}{3}\right) = 0$

$\Rightarrow a \times \frac{4}{9} + 7 \times \frac{2}{3} + b = 0$

$\Rightarrow \frac{4a}{9} + \frac{14}{3} + b = 0 \rightarrow (1)$

Since -3 is a zero of $p(x)$, then $p(-3) = 0$

$\Rightarrow 9a - 21 + b = 0 \rightarrow (2)$

On equating (1) and (2), $\frac{4a}{9} + \frac{14}{3} + \cancel{b} = 9a - 21 + \cancel{b}$

$\Rightarrow \frac{4a}{9} - 9a = -21 - \frac{14}{3}$

$\Rightarrow \frac{4a - 81a}{\cancel{9}3} = \frac{-63 - 14}{\cancel{3}1}$

$\Rightarrow \frac{-\cancel{77}^{1}a}{3} = -\cancel{77}^{1}$

$\Rightarrow a = 3$ //

From eq: (1), $\frac{\cancel{12}^{4}}{\cancel{9}3} + \frac{14}{3} + b = 0 \Rightarrow \frac{\cancel{18}^{6}}{\cancel{3}1} + b = 0$

$\Rightarrow b = -6$ //

9) Let $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Since $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are the zeroes then $(x - (2+\sqrt{3}))$ and $(x - (2-\sqrt{3}))$ are factors.

$\Rightarrow ((x-2) - \sqrt{3})$ and $((x-2) + \sqrt{3})$ are factors

$\Rightarrow (x-2)^2 - 3 = x^2 + 4 - 4x - 3 = x^2 - 4x + 1$ is a factor of $p(x)$.

On dividing $p(x)$ by x^2-4x+1,

$$
\begin{array}{r|l}
 & 2x^2 - x - 1 \\
\hline
x^2-4x+1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\
 & \underset{(-)}{2x^4} \underset{(+)}{-} 8x^3 \underset{(-)}{+} 2x^2 \\
\hline
 & -x^3 + 3x^2 + 3x - 1 \\
 & \underset{(+)}{-x^3} \underset{(-)}{+} 4x^2 \underset{(+)}{-} x \\
\hline
 & -x^2 + 4x - 1 \\
 & \underset{(+)}{-x^2} \underset{(-)}{+} 4x \underset{(+)}{-} 1 \\
\hline
 & 0
\end{array}
$$

Using division algorithm,

$p(x) = (x^2-4x+1)(2x^2-x-1)$

$= (x^2-4x+1)(2x^2-2x+x-1)$

$= (x^2-4x+1)[2x(x-1)+(x-1)]$

$= (x^2-4x+1)(2x+1)(x-1)$

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-1 -2 $<$ 1, -2

$\therefore$ All zeroes of $p(x)$ are $(2+\sqrt{3})$, $(2-\sqrt{3})$, $-\frac{1}{2}$ and 1

10) Let $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of $p(x)$, then

$\left(x-\sqrt{\frac{5}{3}}\right)$ and $\left(x+\sqrt{\frac{5}{3}}\right)$ are the factors of $p(x)$.

Also $x^2 - \frac{5}{3} = \frac{3x^2-5}{3}$ or $3x^2-5$ is a factor of $p(x)$

On dividing $p(x)$ by $3x^2-5$,

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2-5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & \underset{(-)}{3x^4} + 0x^3 \underset{(+)}{-} 5x^2 \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & \underset{(-)}{6x^3} + 0x^2 \underset{(+)}{-} 10x \\
\hline
 & 3x^2 - 5 \\
 & \underset{(-)}{3x^2} \underset{(+)}{-} 5 \\
\hline
 & 0
\end{array}
$$

Using division algorithm,

$p(x) = (3x^2-5)(x^2+2x+1)$

$= (3x^2-5)(x+1)^2 = (3x^2-5)(x+1)(x+1)$

Hence all zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1

11) Let $p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$

$r(x) = -2x + 4$

Using division algorithm, $p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow x^3 - 3x^2 + x + 2 = g(x)(x-2) - 2x + 4$

$\Rightarrow x^3 - 3x^2 + x + 2x + 2 - 4 = g(x)(x-2)$

$\therefore g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{(x-2)}$

$= \underline{\underline{x^2 - x + 1}}$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & \overset{(-)}{x^3} \overset{(+)}{-} 2x^2 \\
\hline
 & -x^2 + 3x - 2 \\
 & \overset{(+)}{-} x^2 \overset{(-)}{+} 2x \\
\hline
 & x - 2 \\
 & \overset{(-)}{x} \overset{(+)}{-} 2 \\
\hline
 & \underline{0}
\end{array}
$$

12) Let $p(x) = 6x^4 + 8x^3 + 5x^2 + ax + b$

On dividing $p(x)$ by $2x^2 - 5$,

$$
\begin{array}{r|l}
 & 3x^2 + 4x + 10 \\
\hline
2x^2 - 5 & 6x^4 + 8x^3 + 5x^2 + ax + b \\
 & \overset{(-)}{6x^4} \overset{(-)}{+} 0x^3 \overset{(+)}{-} 15x^2 \\
\hline
 & 8x^3 + 20x^2 + ax + b \\
 & \overset{(-)}{8x^3} + 0x^2 \overset{(+)}{-} 20x \\
\hline
 & 20x^2 + x(a+20) + b \\
 & \overset{(-)}{20x^2} + 0x \quad \overset{(+)}{-} 50 \\
\hline
 & x(a+20) + (b+50)
\end{array}
$$

Since $p(x)$ is exactly divisible by $2x^2 - 5$, remainder $= 0x + 0$

$a + 20 = 0$

$a = -20$ //

$b + 50 = 0$

$b = -50$ //

13) Let $\alpha = \frac{2}{3}$ and $\beta = -3$ and $p(x) = ax^2 + 7x + b$, which is of the form $Ax^2 + Bx + C$; where $A = a$, $B = 7$, $C = b$

Then $\alpha+\beta = -\frac{B}{A}$

$\Rightarrow \frac{2}{3} - 3 = -\frac{7}{a}$

$\Rightarrow \frac{2-9}{3} = -\frac{7}{a}$

$\Rightarrow \frac{\cancel{-7}}{3} = \frac{\cancel{-7}}{a}$

$\therefore a = 3$

Also, $\alpha\beta = \frac{C}{A}$

$\Rightarrow \frac{2}{\cancel{3}} \times -\cancel{3} = \frac{b}{a}$

$\Rightarrow -2 = \frac{b}{3}$

$\therefore b = -6$

14) Let $p(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$

On dividing $p(x)$ by $x^2 - 2x + k$,

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2-2x+k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & \underline{\cancel{x^4} - 2x^3 + kx^2} \\
 & \cancel{-4x^3} + x^2(16-k) - 25x + 10 \\
 & \underline{\cancel{-4x^3} + 8x^2 - 4kx} \\
 & \cancel{x^2(8-k)} + x(4k-25) + 10 \\
 & \underline{\cancel{x^2(8-k)} - x(16-2k) + k(8-k)} \\
 & x(2k-9) + 10 - 8k + k^2
\end{array}
$$

On comparing the remainder with $1x + a$,

$2k - 9 = 1$

$2k = 10$

$k = 5$

$a = 10 - 8k + k^2$

$= 10 - 40 + 25$

$= -30 + 25$

$a = -5$