13 An honest person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of ₹130. But if he had interchanged amounts invested, he would have received ₹4 more as interest. How much amount did he invest at different rates?

Ans: {₹500, ₹700}

14 If a box containing red and white marbles, half the number of white marbles is equal to one-third the number of red marbles. Thrice the total number of marbles exceeds seven times the number of white marbles by 6. How many marbles of each colour does the box contain? **Ans:** R-18 W-12

15 A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater. **Ans:**{600,400}

16 There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

Ans:{100,80}

17 Draw the graphs of the lines $x = -2$ and $y = 3$. Write the vertices of the figure formed by these lines, the x-axis and the y-axis. Also, find the area of the figure.

Ans:(0,0),(0, 3),(–2,3),(–2,0)

18 Determine, algebraically, the vertices of the triangle formed by the lines $5x - y = 5$, $x + 2y = 1$ and $6x + y = 17$.

Ans:(1,0),(3,-1) (2, 5)

19 It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. How long would it take for each pipe to fill the pool separately? **Ans:**(20, 30)

20 The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father. **Ans:**40 years

A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1 What is the point of intersection of lines represented by $3x - 2y = 6$ and the y-axis? **Ans:**{0,-3}

2 Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not. **Ans:**{No}

3 If $x = a$, $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then find the values of a and b **Ans:**{3,1}

4 If $x = a$, $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then find the values of a and b . **Ans:**{infinite}

5 Solve for x: $99x + 101y = 499$; $101x + 99y = 501$ **Ans:**3 , 2

6 The angles of a cyclic quadrilateral taken in order are $(3y - 5)$, $(4y + 20)$, $(7x + 5)$ and $4x$. Find the angles of the cyclic quadrilateral.

Ans {70,120,110,60}

7 For what value of k the following system of linear equations has a no solution? $x + 2y = 3$ $(k - 1)x + (k + 1)y = k + 2$ **Ans:**{k=3}

8 . Solve for x and y: $3^{x-y} = 27$; $3^{x+y} = 243$ **Ans:**{4, 1}

9 Solve for x and y: $(a + b)x + (a - b)y = a^2 + b^2$

$(a - b)x + (a + b)y = a^2 + b^2$

Ans: {$x= y= \frac{a^2+b^2}{2a}$

10 Determine the values of m and n so that the following system of linear equations have infinite number of solutions:

$(2m - 1)x + 3y - 5 = 0$

$3x + (n - 1)y - 2 = 0$ **Ans :{m=17/4, n=11/5}**

11 Solve for x and y:

$\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2$

Ans: {a^2 , b^2 }

12 Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis.

X HW-7

1) When a line intersects the y-axis, $x=0$

Thus, $3\times0-2y=6$

$-2y=6$

$y=-3$

$\therefore$ The required point is $(0,-3)$

2) $x-3y-2=0$

$-2x+6y-5=0$ be of the form $a_1x+b_1y+c_1=0$
$a_2x+b_2y+c_2=0$; where

$a_1=1, b_1=-3, c_1=-2$

$a_2=-2, b_2=6, c_1=-5$

$\frac{a_1}{a_2}=-\frac{1}{2}$; $\frac{b_1}{b_2}=-\frac{3}{6}=-\frac{1}{2}$; $\frac{c_1}{c_2}=-\frac{2}{5}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$. Thus, the given lines are parallel and do not cross each other.

3) When $x=a, y=b$; $x-y=2 \Rightarrow a-b=2 \rightarrow (1)$

$x+y=4 \Rightarrow a+b=4 \rightarrow (2)$

$(1)+(2)$, $2a=6$

$a=3$

$b=1$ [from eq: (2)]

4) Same as Q.no 3)

5) $99x+101y=499 \rightarrow (1)$

$101x+99y=501 \rightarrow (2)$

$(1)+(2)$, $200x+200y=1000$

$\div200$ $\quad x+y=5 \rightarrow (3)$

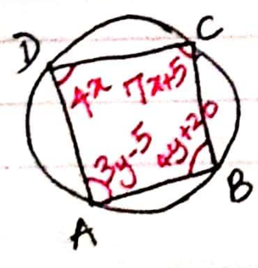
$(1)-(2)$, $-2x+2y=-2$

$\div2$ $\quad -x+y=-1 \rightarrow (4)$

$(3)+(4)$, $2y=4$

$y=2$

From eq: (3), $x=3$

6) 

We know that sum of opposite angles of a cyclic quadrilateral is supplementary

$\angle A + \angle C = 180^\circ \Rightarrow 3y - \cancel{5} + 7x + \cancel{5} = 180^\circ$

$\Rightarrow 3y + 7x = 180^\circ \longrightarrow (1)$

$\angle B + \angle D = 180^\circ \Rightarrow 4y + 20 + 4x = 180^\circ$

$\Rightarrow 4y + 4x = 160^\circ$

$\div 4, \quad y + x = 40^\circ$

$\times 3, \quad 3y + 3x = 120^\circ \longrightarrow (2)$

$(1) - (2), \quad 4x = 60^\circ$

$x = 15^\circ$

$y = 40^\circ - 15^\circ = 25^\circ$

$\therefore$ The angles of the Cyclic quadrilateral are

$\angle A = 3y - 5 = 3 \times 25^\circ - 5 = 75^\circ - 5^\circ = 70^\circ$ //

$\angle C = 180^\circ - \angle A = 180^\circ - 70^\circ = 110^\circ$ //

$\angle B = 4y + 20^\circ = 4 \times 25^\circ + 20^\circ = 100 + 20^\circ = 120^\circ$ //

$\angle D = 180^\circ - \angle B = 180^\circ - 120^\circ = 60^\circ$ //

7) Let $x+2y-3=0$ and $(k-1)x+(k+1)y-(k+2)=0$ be of the form $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$; where

$a_1=1, b_1=2, c_1=-3$

$a_2=k-1, b_2=k+1, c_2=-(k+2)$

For no solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \Rightarrow \underset{\mathrm{I}}{\frac{1}{k-1}}=\underset{\mathrm{II}}{\frac{2}{k+1}}\neq\underset{\mathrm{III}}{\frac{-3}{-(k+2)}}$

From I and II, $k+1=2k-2$

$k-2k=-2-1$

$-k=-3$

$\therefore \underline{k=3}$

8) $3^{x-y}=27 \Rightarrow 3^{x-y}=3^3$

$\therefore x-y=3 \rightarrow (1)$

$3^{x+y}=243 \Rightarrow 3^{x+y}=3^5$

$\therefore x+y=5 \rightarrow (2)$

$(1)+(2),\ 2x=8$

$x=4$ //

From eq.(2), $y=1$ //

~~9) $(a+b)x+(a-b)y=a^2+b^2 \longrightarrow (1)$~~

~~$(a-b)x+(a+b)y=a^2+b^2 \longrightarrow (2)$~~

~~$(1)\times(a+b)$, $(a+b)^2x+(a+b)(a-b)y=a^2+b^2$~~

~~$(2)\times(a-b)$~~

9) $(a+b)x+(a-b)y=a^2+b^2 \longrightarrow (1)$

$(a-b)x+(a+b)y=a^2+b^2 \longrightarrow (2)$

$(1)+(2)$, $(a+\cancel{b}+a-\cancel{b})x+(a-\cancel{b}+a+\cancel{b})y=a^2+b^2+a^2+b^2$

$\Rightarrow 2ax+2ay=2a^2+2b^2$

$\div 2 \Rightarrow ax+ay=a^2+b^2 \longrightarrow (3)$

$(1)-(2)$, $(\cancel{a}+b-\cancel{a}+b)x+(\cancel{a}-b-\cancel{a}-b)y=\cancel{a^2}+\cancel{b^2}-\cancel{a^2}-\cancel{b^2}$

$\Rightarrow 2bx-2by=0$

$\Rightarrow bx-by=0 \rightarrow (4)$

$(3)\times b$, $abx+aby=a^2b+b^3$

$(4)\times a$, $\underline{abx-aby=0}$

$(+)$, $2abx=a^2b+b^3$

$2a\cancel{b}x=\cancel{b}(a^2+b^2)$

$x=\frac{a^2+b^2}{2a}$

Scanned with CamScanner

From eq: (4), $b\frac{(a^2+b^2)}{2a} - by = 0$

$$\cancel{+}\cancel{b}y = \cancel{+}\cancel{b}\frac{(a^2+b^2)}{2a}$$

$$y = \frac{a^2+b^2}{2a}$$

10) Let the given equations $(2m-1)x+3y-5=0$; $3x+(n-1)y-2=0$ be of the form $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$;

$a_1 = 2m-1$, $b_1 = 3$, $c_1 = -5$

$a_2 = 3$, $b_2 = (n-1)$, $c_2 = -2$

For infinite number of solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \underset{\mathrm{I}}{\frac{2m-1}{3}} = \underset{\mathrm{II}}{\frac{3}{n-1}} = \underset{\mathrm{III}}{\frac{-5}{-2}}$$

For I and III, $\frac{2m-1}{3} = \frac{5}{2} \Rightarrow 4m-2 = 15$

$\Rightarrow 4m = 17$

$m = \frac{17}{4}$

For II and III, $\frac{3}{n-1} = \frac{5}{2} \Rightarrow 6 = 5n-5$

$\Rightarrow 5n = 11$

$n = \frac{11}{5}$

11) $\frac{x}{a} + \frac{y}{b} = a+b \Rightarrow bx+ay = ab(a+b) \rightarrow (1)$

$\frac{x}{a^2} + \frac{y}{b^2} = 2 \Rightarrow b^2x+a^2y = 2a^2b^2 \rightarrow (2)$

(1) × a, $abx + a^2y = a^2b(a+b) \rightarrow (3)$

(2), $b^2x + a^2y = 2a^2b^2 \rightarrow (4)$

(3)−(4), $(ab-b^2)x = a^3b + a^2b^2 - 2a^2b^2$

$b(a-b)x = a^3b - a^2b^2$

$\cancel{b}\cancel{(a-b)}x = a^2\cancel{b}\cancel{(a-b)}$

$x = a^2$

From eq:(2), $b^2a^2 + a^2y = 2a^2b^2$

$$a^2y = 2a^2b^2 - a^2b^2$$

$$\cancel{a^2}y = \cancel{a^2}b^2$$

$$y = b^2$$

12) $x - y + 2 = 0 \Rightarrow y = x + 2$

x	0	-2	1
y	2	0	3

$4x - y - 4 = 0 \Rightarrow y = 4x - 4$

x	0	1	2
y	-4	0	4

(graph)

13) Let the person invested ₹x at the rate of 12% simple interest and ₹y at the rate of 10% simple interest.

$$S.I = \frac{PRT}{100}$$

ATQ, $\frac{12x}{100} + \frac{10y}{100} = 130 \Rightarrow 12x + 10y = 13000 \rightarrow (1)$

$\frac{10x}{100} + \frac{12y}{100} = 134 \Rightarrow 10x + 12y = 13400 \rightarrow (2)$

(1)+(2), $22x + 22y = 26400$

$\div 22$, $x + y = 1200 \rightarrow (3)$

(1)−(2), $2x - 2y = -400$

$\div 2$, $x - y = -200 \rightarrow (4)$

(3)+(4), $2x = 1000$

$$x = 500$$

$$y = 700 \quad \text{[from eq:(3)]}$$

Hence the person invested ₹500 at the rate of 12% per year and ₹700 at the rate of 10% per year.

14) Let the no. of red marbles be x and no. of white marbles be y.

ATQ $\frac{1}{2}y = \frac{1}{3}x \Rightarrow 3y = 2x \Rightarrow 2x - 3y = 0 \rightarrow (1)$

Also, $3(x+y) - 7y = 6 \Rightarrow 3x + 3y - 7y = 6$

$\Rightarrow 3x - 4y = 6 \rightarrow (2)$

(1) × 4, $8x - 12y = 0$

(2) × 3, $9x - 12y = 18$

(−) $-x = -18$

$x = 18$

From eq: (1), $2 \times 18 - 3y = 0$

$-3y = -36$

$y = 12$

Hence the no. of red marbles = 18

no. of white marbles = 12

15) Let the Cost price of saree be ₹x and list price of the sweater be ₹y

ATQ, S.P of saree $= x + 8\%$ of $x = x + \frac{8}{100}x = \frac{108x}{100}$

S.P of sweater $= y - 10\%$ of $y = y - \frac{10}{100}y = \frac{90y}{100}$

Thus $\frac{108x}{100} + \frac{90y}{100} = 1008$

$\Rightarrow 108x + 90y = 100800 \longrightarrow (1)$

Also, $\frac{110x}{100} + \frac{92y}{100} = 1028$

$\Rightarrow 110x + 92y = 102800 \rightarrow (2)$

S.P of saree $= x + \frac{10}{100}x = \frac{110x}{100}$

S.P of sweater $= y - 8\%$ of y

$= y - \frac{8}{100}y$

$= \frac{92y}{100}$

(1) − (2), $-2x - 2y = -2000$

÷ 2, $-x - y = -1000$

$x + y = 1000$

$y = 1000 - x \rightarrow (3)$

Substituting (3) in (1),

$108x + 90(1000 - x) = 100800$

$108x + 90000 - 90x = 100800$

$18x = 10800$

$x = \frac{10800}{18} = \underline{\underline{600}}$

$y = 1000 - 600 = \underline{\underline{400}}$

Hence the Cost price of the saree = ₹600
list price of sweater = ₹400

16)

A	B
x	y
$x-10$	$y+10$
$x+20$	$y-20$

Let the no. of students in two halls be x and y

ATQ, $x-10 = y+10 \Rightarrow x-y = 20 \rightarrow (1)$

Also, $x+20 = 2(y-20)$, $x+20 = 2y-40 \Rightarrow x-2y = -60 \rightarrow (2)$

$(1)-(2)$, $y = 80$

from eq:(1), $x = 100$

Hence the no. of students in the two halls = 100 and 80

17) graph

18) $5x-y=5 \rightarrow (1)$

$x+2y=1 \rightarrow (2)$

$6x+y=17 \rightarrow (3)$

From eq:s (1) and (2),

$2\times(1)$, $10x-2y = 10$

(2), $x+2y = 1$

$(+)$, $11x = 11$

$x = 1$

$y = 0$

Hence the point of intersection of lines I and II is $(1,0)$

From eq:s (2) and (3),

$x+2y = 1$

$(3)\times 2$, $12x+2y = 34$

$(-)$, $-11x = -33$

$x = 3$

$y = -1$

Hence the point of intersection of lines II and III is $(3,-1)$

From eq:s (1) and (3),

$5x-y=5$

$6x+y=17$

$(+)$, $11x = 22$

$x = 2$

$y = 5$

Hence the point of intersection of lines I and III is $(2,5)$

Thus the vertices of the Δ formed by the given lines are $(1,0)$, $(3,-1)$ and $(2,5)$

19) Let the time taken by larger pipe alone to fill the tank be x hrs

Part of tank filled by the larger pipe in 1 hr $= \frac{1}{x}$

$\therefore$ Part of tank filled by the larger pipe in 12 hrs $= \frac{12}{x}$

Similarly part of tank filled by small pipe in 12 hrs $= \frac{12}{y}$

Together they filled the tank.

ATQ, $\frac{12}{x} + \frac{12}{y} = 1 \rightarrow (1)$

Also, $\frac{4}{x} + \frac{9}{y} = \frac{1}{2} \rightarrow (2)$

Put $\frac{1}{x} = a$; $\frac{1}{y} = b$

$12a + 12b = 1 \rightarrow (3)$

$4a + 9b = \frac{1}{2} \rightarrow (4)$ X3

$\Rightarrow 12a + 27b = \frac{3}{2} \rightarrow (5)$

$(3)-(5)$, $-15b = 1 - \frac{3}{2} = \frac{-1}{2}$

$b = \frac{1}{30} \Rightarrow \underline{y = 30}$

From eq: (3), $12a + \frac{\cancel{12}^{2}}{\cancel{30}_{5}} = 1$

$12a = 1 - \frac{2}{5} = \frac{3}{5}$

$a = \frac{1}{20} \Rightarrow \underline{x = 20}$

Hence the time taken by larger and smaller pipes are 20 hrs and 30 hrs respt.

20) Let the age of father be x. and sum of ages of two children be y

ATQ, $x = 2y$

$x - 2y = 0 \longrightarrow (1)$

After 20 yrs, age of father $= x + 20$

age of 2 children $= y + 40$

Then, $x + 20 = y + 40$

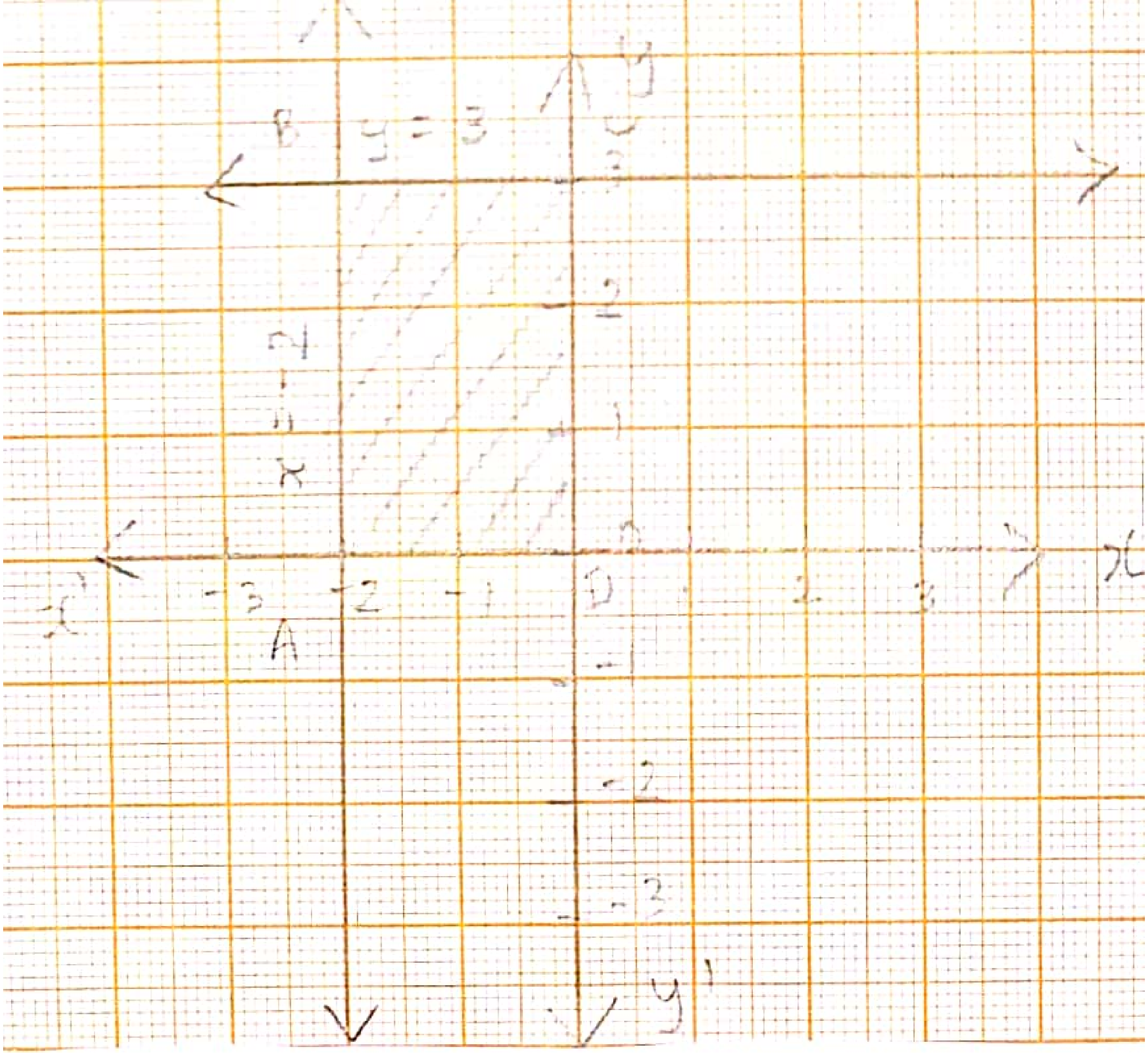
$x - y = 20 \longrightarrow (2)$

(1) − (2), $-y = -20$

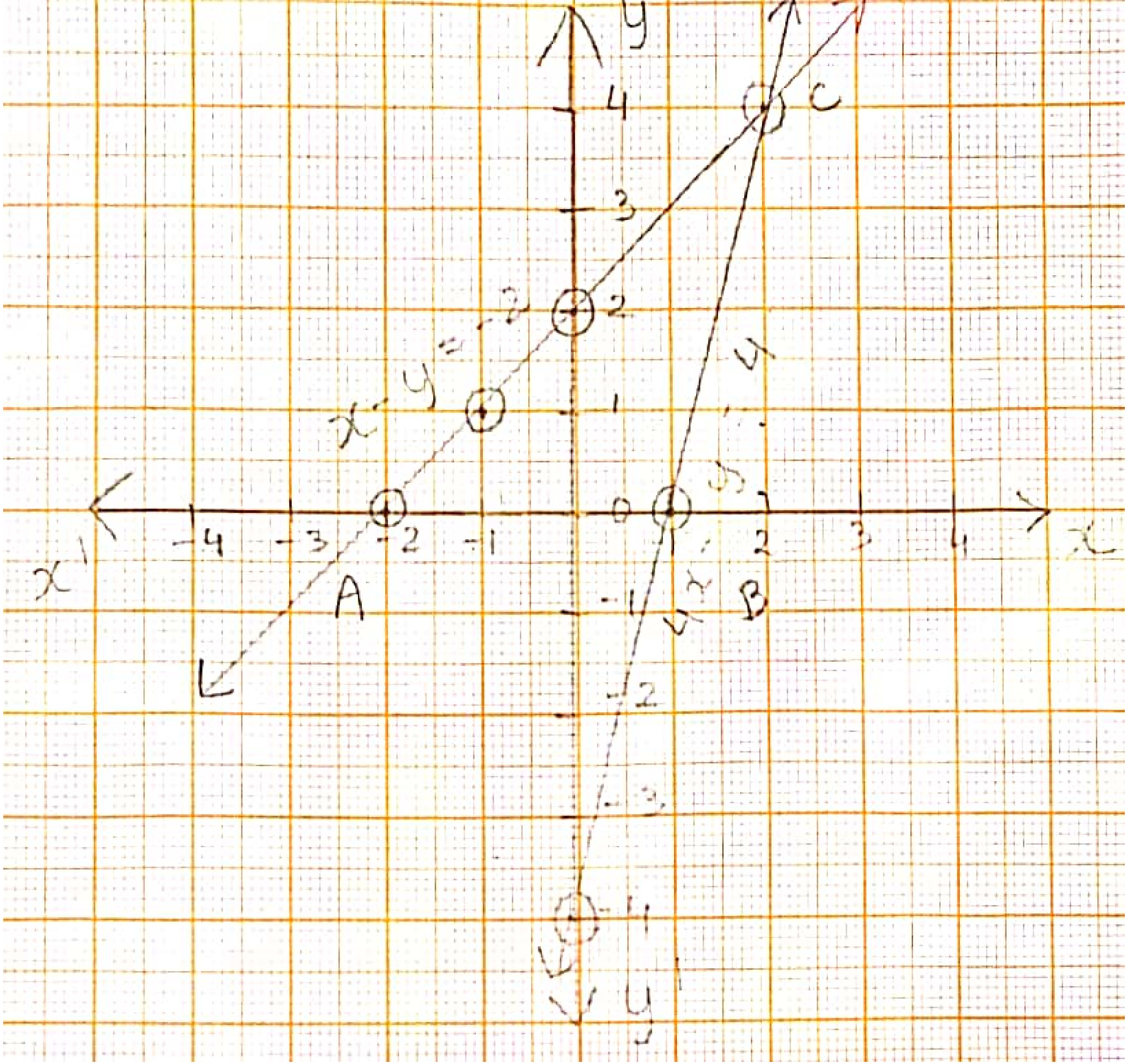
$y = 20$

From eq: (2), $x = 40$

Hence the age of father $=$ 40 years.



y
B
y = 3
x = -2
A
x'
x
y'
-3
-2
-1
O
2
3
1
2
-1
-2
-3



y
C
4
3
2
1
0
x - y = -2
4x - y = 4
x'
-4
-3
-2
-1
2
3
4
x
A
B
-1
-2
-3
-4
y'