X Homework-5 (REAL NUMBERS)

One mark Questions

1) State Euclid's Division Lemma.
2) Identify the rational number: $5-\sqrt{3}, 5+\sqrt{3}, 4+\sqrt{2}, 5+\sqrt{9}$
3) Has the rational number $\frac{441}{2^2 \times 5^7 \times 7^2}$ a terminating or non-terminating decimal expansion?
4) Has the rational number $\frac{51}{1500}$ a terminating or non-terminating decimal expansion?
5) Write whether the rational number $\frac{7}{75}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.
6) What is the HCF of smallest prime number and the smallest composite number?
7) Find after how many places of decimal the decimal form of the number will $\frac{27}{2^3 \cdot 5^4 \cdot 3^2}$ terminate?
8) Express 429 as product of its prime factors.

Two marks Questions

9) Check whether 6^n can never end with 0 for any natural number n.
10) If two positive integers p and q are written as $p = a^2 b^3$ and $q = a^3 b$; a, b are prime numbers, then verify $LCM(p,q) \times HCF(p,q) = pq$
11) Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number.
12) If HCF of 65 and 117 is expressible in the form of $65n - 117$ then find the value of n.
13) On a morning walk three persons step out together and their steps measure 30cm, 36cm and 40cm respt. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

3 marks Questions

14) Show that the square of any positive odd integer is of the form $8m+1$, for some integer m.

15) Prove that $7+3\sqrt{2}$ is not a rational number.

16) An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

17) Find HCF and LCM of 306 and 54 and verify that HCF×LCM = product of the two given numbers.

18) Prove that $\frac{2\sqrt{3}}{5}$ is irrational number.

19) Prove that $2-3\sqrt{5}$ is irrational number.

20) Show that exactly one of the numbers $n, n+2$ or $n+4$ is divisible by 3.

21) Find HCF and LCM of 404 and 96 and verify that HCF×LCM = product of the two given numbers.

22) Prove that $\sqrt{3}$ is an irrational number.

23) Find the largest number which on dividing 1251, 9377 and 15628 leaves remainders 1, 2 and 3 respectively.

X Homework-5 (Solutions)

1) Euclid's Division Lemma states that for any two positive integers a and b $(a>b)$, there exists two unique integers q and r such that $a=bq+r$; $0 \le r < b$.
i.e. dividend = divisor × quotient + remainder.

2) $5+\sqrt{9}=5+3$, a rational number.

3) $\frac{\cancel{441}\ 63\cancel{9}}{2^2\times5^7\times\cancel{7^2}\ 1} = \frac{9}{2^2\times5^7}$, since the denominator is of the form $2^m\times5^n$; where m and n are non-negative integers, it has a terminating decimal expansion.

4) $\frac{\cancel{51}\ 17}{\cancel{1500}\ 500} = \frac{17}{500} = \frac{17}{5^3\times2^2}$

$$\begin{array}{r|l} 5 & 500 \\ 5 & 100 \\ 5 & 20 \\ 2 & 4 \\ & 2 \end{array}$$

it has a terminating decimal expansion since the denominator is of the form $2^m\times5^n$; where m and n are non-negative integers.

5) $\frac{7}{75} = \frac{7}{5^2\times3}$

$$\begin{array}{r|l} 5 & 75 \\ 5 & 15 \\ & 3 \end{array}$$

it has a non-terminating repeating decimal expansion since the denominator is not of the form $2^m\times5^n$; where m and n are non-negative integers.

6) smallest prime number = 2
smallest composite number = $4 = 2^2$
HCF $(2,4) = \underline{\underline{2}}$

7) $\frac{\cancel{27}\ 3}{2^3\times5^4\times\cancel{3^2}\ 1} = \frac{3\times2}{2^4\times5^4} = \frac{6}{10^4} = 0.0006$ //

Hence the decimal expansion terminates after 4 decimal places.

8) $429 = 3\times11\times13$

$$\begin{array}{r|l} 3 & 429 \\ 11 & 143 \\ & 13 \end{array}$$

9) $6^n = (2\times3)^n = 2^n\times3^n$
Since the prime factorisation of 6^n does not contain 5, it does not end with digit 0. According to fundamental theorem of arithmetic no other prime factors exist in the prime factorisation of 6^n. Hence there is no natural number

n for which 6^n ends with digit 0.

10) $p = a^2b^3$

$q = a^3b$

$LCM(p,q) = a^3b^3$

$HCF(p,q) = a^2b$

Verification:

LHS, $LCM(p,q) \times HCF(p,q) = a^3b^3 \times a^2b = a^5b^4$

RHS, $pq = a^2b^3 \times a^3b = a^5b^4$

$\therefore$ LHS = RHS. Hence verified.

11) Let us assume that $5+3\sqrt{2}$ is a rational number.

Then, $5+3\sqrt{2} = \frac{a}{b}$; where a and b are integers and $b \neq 0$

$$\Rightarrow 3\sqrt{2} = \frac{a}{b} - 5 = \frac{a-5b}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a-5b}{3b}$$; which is a rational number since a and b are integers.

Thus $\sqrt{2}$ is also a rational number. But this contradicts the fact that $\sqrt{2}$ is an irrational number (given). This contradiction arises due to our wrong assumption that $5+3\sqrt{2}$ is a rational number. Hence $5+3\sqrt{2}$ is irrational.

12) $117 > 65$

Using Euclid's division algorithm,

$117 = 65 \times 1 + 52$

$65 = 52 \times 1 + 13$

$54 = 13 \times 4 + 0$

$65) 117 (1$
65
$52) 65 (1$
52
$13) 54 (4$
54
0

$\therefore HCF(117, 65) = 13.$

$\therefore 65n - 117 = 13$

$65n = 130$

$$n = \frac{130}{65} = \underline{\underline{2}}$$

13) $30 = 3 \times 2 \times 5$

$36 = 3^2 \times 2^2$

$40 = 2^3 \times 5$

$LCM(30, 36, 40)$

$= 3^2 \times 2^3 \times 5$

$= 360$ cm

$3 \,|\, 30$, $2 \,|\, 10$, 5

$3 \,|\, 36$, $3 \,|\, 12$, $2 \,|\, 4$, 2

Hence the minimum distance they walked to cover same distance in complete steps $= 360$ cm $= 3.6$ m//

14) According to Euclid's division lemma, any positive odd integer is of the form $4q+1$ or $4q+3$ for some integer q.

Case 1:- When $a = 4q+1$

$$a^2 = (4q+1)^2$$
$$= 16q^2+8q+1 = 8(2q^2+q)+1 = 8m+1;$$
where $m = 2q^2+q$

Case 2:- when $a = 4q+3$

$$a^2 = (4q+3)^2 = 16q^2+24q+9$$
$$= (16q^2+24q+8)+1$$
$$= 8(2q^2+3q+1)+1 = 8m+1;$$
Where $m = 2q^2+3q+1$

Hence square of any positive odd integer is of the form $8m+1$ for some integer m.

15) Let us assume $\sqrt{2}$ is a rational number.

Then $\sqrt{2} = \frac{p}{q}$; where p and q are coprime numbers and $q \neq 0$.

Squaring on both sides $\Rightarrow 2 = \frac{p^2}{q^2}$

$\Rightarrow 2q^2 = p^2 \longrightarrow (1)$

$\Rightarrow$ 2 divides p^2

$\Rightarrow$ 2 divides p

Let $p = 2c$

From eq: (1), $2q^2 = 4c^2$

$\Rightarrow q^2 = 2c^2$

$\Rightarrow$ 2 divides q^2

$\Rightarrow$ 2 divides q

Thus 2 is a common factor for p and q. But this contradicts the fact that p and q are coprime numbers. Thus our assumption is wrong and $\sqrt{2}$ is an irrational no.

Again, let us assume that $7+3\sqrt{2}$ is a rational number.

Then, $7+3\sqrt{2} = \frac{a}{b}$; where a and b are integers and $b \neq 0$.

$\Rightarrow 3\sqrt{2} = \frac{a}{b} - 7 = \frac{a-7b}{b}$

$\Rightarrow \sqrt{2} = \frac{a-7b}{3b}$, which is a rational number since a and b are integers.

Thus $\sqrt{2}$ is also a rational number. But this contradicts the fact that $\sqrt{2}$ is an irrational number (proved above). Thus our assumption is wrong and hence $7+3\sqrt{2}$ is not a rational number.

16) $616 > 32$

Using Euclid's division algorithm,

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

$$\begin{array}{r|l} 32 & 616 \,(19 \\ & \underline{608} \\ 8 & 32 \,(4 \\ & \underline{32} \\ & 0 \end{array}$$

$\therefore$ HCF$(616, 32) = \underline{\underline{8}}$ members in 1 row

Hence required no. of columns $= \underline{8}$

17) $306 = 3^2 \times 2 \times 17$

$54 = 3^3 \times 2$

$$\begin{array}{r|l} 3 & 306 \\ 3 & 102 \\ 2 & 34 \\ & 17 \end{array} \qquad \begin{array}{r|l} 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

HCF$(306, 54) = 3^2 \times 2 = 18$

LCM$(306, 54) = 3^3 \times 2 \times 17 = 918$

$\therefore$ LCM $\times$ HCF $= 18 \times 918 = 16524$

Product of numbers $= 306 \times 54 = 16524$

$\therefore$ LCM $\times$ HCF $=$ product of numbers.

18) Prove $\sqrt{3}$ is an irrational no. same as Q.No 15

Again, let us assume $\frac{2\sqrt{3}}{5}$ is a rational number.

Then $\frac{2\sqrt{3}}{5} = \frac{a}{b}$; where a and b are integers and $b \neq 0$

$\Rightarrow \sqrt{3} = \frac{5a}{2b}$, which is a rational number since a and b are integers

Thus $\sqrt{3}$ is also a rational number. This contradicts the fact that $\sqrt{3}$ is an irrational number. Thus our assumption is wrong and hence $\frac{2\sqrt{3}}{5}$ is an irrational number.

19) prove $\sqrt{5}$ is an irrational number same as Q.No 15

Again, let us assume $2-3\sqrt{5}$ is a rational number.

Then, $2-3\sqrt{5} = \frac{a}{b}$; where a and b are integers and $b \neq 0$

$$\Rightarrow -3\sqrt{5} = \frac{a}{b} - 2$$

$$\Rightarrow 3\sqrt{5} = 2 - \frac{a}{b} = \frac{2b-a}{b}$$

$$\Rightarrow \sqrt{5} = \frac{2b-a}{3b}$$, which is a rational number since a and b are integers.

Thus $\sqrt{5}$ is also a rational number. But this contradicts the fact that $\sqrt{5}$ is an irrational number. Thus our assumption is wrong and hence $2-3\sqrt{5}$ is an irrational number.

20) According to Euclid's division lemma,

$n = 3q + r$; $0 \leq r < 3$ for some integers n and q.

ie $r = 0, 1, 2$.

Case 1:- when $r = 0$

$n = 3q$, divisible by 3

$n+2 = 3q+2$, not divisible by 3

$n+4 = 3q+4 = (3q+3)+1 = 3(q+1)+1$, not divisible by 3

Case 2:- when $r = 1$

$n = 3q+1$

$n+2 = 3q+3 = 3(q+1)$, divisible by 3

$n+4 = 3q+5 = (3q+3)+2 = 3(q+1)+2$, not divisible by 3

Case 3:- when $r = 2$

$n = 3q+2$, not divisible by 3

$n+2 = 3q+4 = (3q+3)+1 = 3(q+1)+1$, not divisible by 3

$n+4 = 3q+6 = 3(q+2)$, divisible by 3

Hence exactly one of the numbers n, $n+2$ or $n+4$ is divisible by 3.

21) $404 = 2^2 \times 101$

$96 = 2^5 \times 3$

$$\begin{array}{r|l} 2 & 404 \\ \hline 2 & 202 \\ \hline & 101 \end{array} \qquad \begin{array}{r|l} 3 & 96 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$HCF(404, 96) = 2^2 = 4$

$LCM(404, 96) = 2^5 \times 101 \times 3 = 9696$

$\therefore HCF \times LCM = 4 \times 9696 = 38784$

product of no.s $= 404 \times 96 = 38784$

$\therefore HCF \times LCM =$ product of the numbers

22) Let us assume that $\sqrt{3}$ is a rational number.

Then, $\sqrt{3} = \frac{p}{q}$; where p and q are coprime numbers and $q \neq 0$

$\Rightarrow \sqrt{3}q = p$

(Squaring on both sides) $\Rightarrow 3q^2 = p^2 \longrightarrow (1)$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p.

Let $p = 3s$

From eq: (1), $3q^2 = 9s^2$

$\Rightarrow q^2 = 3s^2$

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ 3 divides q

Thus 3 is a common factor of p and q. But this contradicts the fact that p and q are co-prime numbers. Thus our assumption is wrong and hence $\sqrt{3}$ is an irrational number.

23) $1251 - 1 = 1250$

$9377 - 2 = 9375$

$15628 - 3 = 15625$

$15625 > 9375$

$3125 > 1250$

$$\begin{array}{l} 9375)\,15625\,(1 \\ \quad\;\; 9375 \\ \quad\;\; 6250)\,9375\,(1 \\ \qquad\qquad 6250 \\ \qquad\qquad 3125)\,6250\,(2 \\ \qquad\qquad\qquad 6250 \\ \qquad\qquad\qquad\;\; 0 \end{array}$$

Using Euclid's division algorithm,

$$\begin{array}{l} 1250)\,3125\,(2 \\ \quad\;\; 2500 \\ \quad\;\; 625)\,1250\,(2 \\ \qquad\quad 1250 \\ \qquad\qquad 0 \end{array}$$

$15625 = 9375 \times 1 + 6250$

$9375 = 6250 \times 1 + 3125$

$6250 = 3125 \times 2 + 0$

$3125 = 1250 \times 2 + 625$

$1250 = 625 \times 2 + 0$

$\therefore HCF(1250, 9375, 15625) = 625$

Hence the required largest number is 625