X Homework-2 (Polynomials)

1) If $p(x) = g(x).q(x) + r(x)$, degree of $p(x) = 6$, degree of $g(x) = 3$, then degree of $q(x)$ is (a) 2 (b) 3 (c) 1 (d) 4

2) If $p(x) = g(x).q(x) + r(x)$, then degree of $q(x)$ is always less than (a) the degree of $g(x)$ (b) the degree of $p(x)$ (c) equal to degree of $p(x)$ (d) the degree of $r(x)$.

3) If $(x+1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a - 3b = 4$
(a) $a = -1, b = -2$ (b) $a = 2, b = 5$ (c) $a = 5, b = 2$ (d) $a = 2, b = 0$

4) If $4x^2 - 6x - m$ is divisible by $x - 3$, the value of m is exact divisor of (a) 9 (b) 45 (c) 20 (d) 36.

5) A polynomial of degree five is divided by a quadratic polynomial. If it leaves a remainder, then find the degree of remainder.

6) If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, then the remainder comes out to be $ax + b$. Find the values of a and b.

7) Find all zeroes of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$ if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

8) Obtain all zeroes of $3x^4 - 15x^3 + 13x^2 + 25x - 30$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

9) On dividing $x^4 - x^3 - 3x^2 + 3x + 2$ by a polynomial $g(x)$, the quotient and remainder are $x^2 - x + 2$ and $2x$ respectively. Find $g(x)$

10) If α, β and γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$!

11) Show that $\frac{1}{2}$ and $-\frac{3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial.

12) Find the value of p for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

13) If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

X Homework-2 (POLYNOMIALS) – Answers

1) degree $(q(x)) = 6-3 = 3$ (b)

2) equal to degree of $p(x)$ (c)

3) Let $p(x) = 2x^3 + ax^2 + 2bx + 1$, then $p(-1) = 0$

$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$\Rightarrow -2 + a - 2b + 1 = 0$

$\Rightarrow a - 2b = 1$

$2\times \Rightarrow 2a - 4b = 2 \longrightarrow (1)$

Given, $2a - 3b = 4 \longrightarrow (2)$

$(1)-(2)$, $-4b + 3b = 2 - 4$

$-b = -2$

$b = 2$

From eq: (1), $a - 4 = 1$

$a = 5$ (c)

4) Let $p(x) = 4x^2 - 6x - m$

$p(3) = 0$

$\Rightarrow 4(3)^2 - 6(3) - m = 0$

$\Rightarrow 36 - 18 - m = 0$

$\Rightarrow m = 18$

18 is exact divisor of 36 (d)

5) Degree of remainder will be less than the divisor

i.e, less than 2

Hence degree is 1 or 0.

6) Let $p(x) = 6x^4 + 8x^3 + 17x^2 + 21x + 7$

On dividing $p(x)$ by $3x^2 + 4x + 1$,

$$
\begin{array}{r|l}
 & 2x^2 + 5 \\
\hline
3x^2 + 4x + 1 & \cancel{6x^4} + \cancel{8x^3} + 17x^2 + 21x + 7 \\
 & \underset{(-)}{}\cancel{6x^4} \underset{(-)}{+} \cancel{8x^3} \underset{(-)}{+} 2x^2 \\
\hline
 & 15x^2 + 21x + 7 \\
 & \underset{(-)}{}\cancel{15x^2} \underset{(-)}{+} 20x \underset{(-)}{+} 5 \\
\hline
 & x + 2
\end{array}
$$

Thus remainder $= x + 2$.

On comparing the remainder $x+2$ with $ax+b$, $\boxed{\begin{array}{l} a = 1 \\ b = 2 \end{array}}$

7) Let $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Since $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of $p(x)$,

$[x-(2+\sqrt{3})]$ and $[x-(2-\sqrt{3})]$ are the factors of $p(x)$.

Then $((x-2)-\sqrt{3})$ and $((x-2)+\sqrt{3})$ are also factors.

Consequently, $(x-2)^2 - (\sqrt{3})^2 = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is also a factor of $p(x)$

On dividing $p(x)$ by $g(x) = x^2 - 4x + 1$,

$$
\begin{array}{r}
2x^2 - x - 1 \\
x^2-4x+1 \overline{\smash{\big)}\, 2x^4 - 9x^3 + 5x^2 + 3x - 1} \\
\underset{(-)}{}\cancel{2x^4} \underset{(+)}{-} 8x^3 \underset{(-)}{+} 2x^2 \qquad\qquad \\
\hline
-\cancel{x^3} + 3x^2 + 3x - 1 \\
\underset{(+)}{-}\cancel{x^3} \underset{(-)}{+} 4x^2 \underset{(+)}{-} x \qquad \\
\hline
-\cancel{x^2} + \cancel{4x} - \cancel{1} \\
\underset{(+)}{-}\cancel{x^2} \underset{(-)}{+} \cancel{4x} \underset{(+)}{-} \cancel{1} \\
\hline
0
\end{array}
$$

$\therefore$ quotient, $q(x) = 2x^2 - x - 1$

remainder, $r(x) = 0$

Using division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$= (x^2 - 4x + 1)(2x^2 - x - 1)$

$= (x^2 - 4x + 1)(2x^2 - 2x + x - 1)$ S P -1 -2 $<$ 1 -2

$= (x^2 - 4x + 1)[2x(x-1) + (x-1)]$

$= (x^2 - 4x + 1)(2x+1)(x-1)$

$\therefore$ All zeroes of $p(x)$ are $2+\sqrt{3}, 2-\sqrt{3}, -\frac{1}{2}$ and 1.

8) Let $p(x) = 3x^4 - 15x^3 + 13x^2 + 25x - 30$

Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of $p(x)$,

$\left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are factors of $p(x)$.

Then $x^2 - \frac{5}{3}$ is also a factor of $p(x)$.

Consequently $3x^2 - 5$ is also a factor of $p(x)$.

On dividing $p(x)$ by $g(x) = 3x^2 - 5$,

$$
\begin{array}{r|l}
 & x^2 - 5x + 6 \\
\hline
3x^2 - 5 & \cancel{3x^4} - 15x^3 + 13x^2 + 25x - 30 \\
 & \overset{(-)}{\cancel{3x^4}} + 0x^3 \overset{(+)}{-} 5x^2 \\
\hline
 & -\cancel{15x^3} + 18x^2 + \cancel{25x} - 30 \\
 & \overset{(+)}{-}\cancel{15x^3} + 0x^2 \overset{(-)}{+} \cancel{25x} \\
\hline
 & \cancel{18x^2} - \cancel{30} \\
 & \overset{(-)}{\cancel{18x^2}} \overset{(+)}{-} \cancel{30} \\
\hline
 & 0
\end{array}
$$

$\therefore$ quotient, $q(x) = x^2 - 5x + 6$

remainder, $r(x) = 0$

Using division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$= (3x^2 - 5)(x^2 - 5x + 6)$

$= (3x^2 - 5)(x - 3)(x - 2)$

S	P	
-5	6	$< \begin{matrix} -3 \\ -2 \end{matrix}$

Hence all zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, 3 and 2.

9) Let $p(x) = x^4 - x^3 - 3x^2 + 3x + 2$, $q(x) = x^2 - x - 2$ and $r(x) = 2x$.

Using division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow x^4 - x^3 - 3x^2 + 3x + 2 = g(x)(x^2 - x - 2) + 2x$

$\Rightarrow x^4 - x^3 - 3x^2 + 3x + 2 - 2x = g(x)(x^2 - x - 2)$

$\Rightarrow (x^2 - x - 2)(g(x)) = x^4 - x^3 - 3x^2 + x + 2$

$\therefore g(x) = \dfrac{x^4 - x^3 - 3x^2 + x + 2}{x^2 - x - 2}$

On dividing,

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^2 - x - 2 & \cancel{x^4} - \cancel{x^3} - 3x^2 + x + 2 \\
 & \overset{(-)}{\cancel{x^4}} \overset{(+)}{-} \cancel{x^3} \overset{(+)}{-} 2x^2 \\
\hline
 & -\cancel{x^2} + \cancel{x} + \cancel{2} \\
 & \overset{(+)}{-}\cancel{x^2} \overset{(-)}{+} \cancel{x} \overset{(-)}{+} \cancel{2} \\
\hline
 & 0
\end{array}
$$

$\therefore g(x) = \underline{\underline{x^2 - 1}}$

10) Let $p(x) = 6x^3 + 3x^2 - 5x + 1$ be of the form $ax^3 + bx^2 + cx + d$ where $a = 6, b = 3, c = -5, d = 1$.

Then $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-5}{6}$

$\alpha\beta\gamma = \frac{-d}{a} = \frac{-1}{6}$

$$\therefore \frac{1\times\beta\gamma}{\alpha\times\beta\gamma}+\frac{1\times\alpha\gamma}{\beta\times\alpha\gamma}+\frac{1\times\alpha\beta}{\gamma\times\alpha\beta} = \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{-5}{6} \div \frac{-1}{6} = \frac{-5}{6} \times -\frac{6}{1} = \underline{\underline{5}}$$

11) Let $p(x) = 4x^2+4x-3$ be of the form ax^2+bx+c; $a=4$, $b=4$, $c=-3$.

$$p\left(\tfrac{1}{2}\right) = 4\left(\tfrac{1}{2}\right)^2 + 4\left(\tfrac{1}{2}\right) - 3 = \cancel{4}^{1} \times \frac{1}{\cancel{4}_{1}} + \frac{\cancel{4}^{2}}{\cancel{2}_{1}} - 3 = 1+2-3 = 3-3 = \underline{\underline{0}}$$

$$p\left(\tfrac{-3}{2}\right) = 4\left(\tfrac{-3}{2}\right)^2 + 4\left(\tfrac{-3}{2}\right) - 3 = \cancel{4}^{1} \times \frac{9}{\cancel{4}_{1}} - \frac{\cancel{12}^{6}}{\cancel{2}_{1}} - 3 = 9-6-3 = 9-9 = \underline{\underline{0}}$$

Thus $\frac{1}{2}$ and $-\frac{3}{2}$ are zeroes of $p(x)$.

Verification:-

Let $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$ be the zeroes.

Sum of zeroes $= \alpha+\beta = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2} = -\frac{4}{4} = \frac{-b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$.

Product of zeroes $= \alpha\beta = \frac{1}{2} \times -\frac{3}{2} = -\frac{3}{4} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$.

12) Let $px^2 - 14x + 8 = 0$ be in the form $ax^2+bx+c=0$; where $a=p$; $b=-14$ and $c=8$ and α and 6α be the zeroes.

Sum of zeroes $= \alpha + 6\alpha = 7\alpha = \frac{-b}{a} = \frac{14}{p}$

$$\Rightarrow \cancel{7}^{1}\alpha = \frac{\cancel{14}^{2}}{p} \Rightarrow \alpha = \frac{2}{p}$$

Product of zeroes $= \alpha \times 6\alpha = \frac{c}{a}$

$$\Rightarrow 6\alpha^2 = \frac{8}{p} \Rightarrow 6 \times \frac{4}{\cancel{p^2}_{1}} = \frac{8}{\cancel{p}_{1}}$$

$$\Rightarrow 24 = 8p$$

$$\therefore p = \frac{24}{8} = \underline{\underline{3}}$$

13) Let $p(x) = 3x^2+2x+1$ be of the form ax^2+bx+c where $a=3$, $b=2$, $c=1$.

Then $\alpha+\beta = -\frac{b}{a} = -\frac{2}{3}$ //

$\alpha\beta = \frac{c}{a} = \frac{1}{3}$ //

For the new polynomial,

Sum of zeroes $= \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta} = \frac{(1-\alpha)(1+\beta)+(1-\beta)(1+\alpha)}{(1+\alpha)(1+\beta)}$

$= \frac{1+\cancel{\beta}-\cancel{\alpha}-\alpha\beta+1+\cancel{\alpha}-\cancel{\beta}-\alpha\beta}{1+\beta+\alpha+\alpha\beta}$

$= \frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} = \frac{2(1-\alpha\beta)}{1+(\alpha+\beta)+\alpha\beta}$

$= \frac{2\left(1-\frac{1}{3}\right)}{1+\left(-\frac{2}{3}\right)+\frac{1}{3}} = \frac{2\times\frac{2}{3}}{\frac{3-2+1}{3}} = \frac{\frac{4}{\cancel{3}\,1}}{\frac{2}{\cancel{3}\,1}} = \frac{4}{2} = \underline{\underline{2}}$

Product of zeroes $= \frac{(1-\alpha)}{(1+\alpha)}\frac{(1-\beta)}{(1+\beta)} = \frac{1-\beta-\alpha+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$

$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} = \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{3+2+1}{\cancel{3}\,1}}{\frac{3-2+1}{\cancel{3}\,1}}$

$= \frac{6}{2} = \underline{\underline{3}}$

$\therefore$ The required polynomial is $x^2-(\text{Sum of zeroes})x+\text{product of zeroes}$

$= \underline{\underline{x^2-2x+3}}$