X Homework-1 Polynomials

1) The zeroes of the polynomial $x^2 - \sqrt{2}x - 12$ are
(a) $\sqrt{2}, -\sqrt{2}$ (b) $3\sqrt{2}, -2\sqrt{2}$ (c) $-3\sqrt{2}, 2\sqrt{2}$ (d) $3\sqrt{2}, 2\sqrt{2}$

2) The zeroes of the polynomial $4x^2 + 5\sqrt{2}x - 3$ are
(a) $-3\sqrt{2}, \sqrt{2}$ (b) $-3\sqrt{2}, \frac{\sqrt{2}}{2}$ (c) $\frac{-3\sqrt{2}}{2}, \frac{\sqrt{2}}{4}$ (d) none of these

3) The zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$ are
(a) $-3, 4$ (b) $-\frac{3}{2}, \frac{4}{3}$ (c) $-\frac{4}{3}, \frac{3}{2}$ (d) none of these.

4) The zeroes of the polynomial $7x^2 - \frac{11}{3}x - \frac{2}{3}$ are
(a) $2/3, -1/7$ (b) $2/7, -1/3$ (c) $-2/3, 1/7$ (d) none of these

5) The sum and the product of the zeroes of a quadratic polynomial are 3 and -10 respt. The quadratic polynomial is
(a) $x^2 - 3x + 10$ (b) $x^2 + 3x - 10$ (c) $x^2 - 3x - 10$ (d) $x^2 + 3x + 10$

6) A quadratic polynomial whose zeroes are $\frac{3}{5}$ and $-\frac{1}{2}$ is
(a) $10x^2 + x + 3$ (b) $10x^2 + x - 3$ (c) $10x^2 - x + 3$ (d) $10x^2 - x - 3$

7) The zeroes of the quadratic polynomial $x^2 + 88x + 125$ are
(a) both positive (b) both negative (c) one positive one negative (d) both equal

8) If α and β are the zeroes of $x^2 + 5x + 8$, then the value of $\alpha + \beta$ is (a) 5 (b) -5 (c) 8 (d) -8

9) If α and β are the zeroes of $2x^2 + 5x - 9$, then the value of $\alpha\beta$ is (a) $-5/2$ (b) $5/2$ (c) $-9/2$ (d) $9/2$

10) If -2 and 3 are the zeroes of $x^2 + (a+1)x + b$, then
(a) $a = -2, b = 6$ (b) $a = 2, b = -6$ (c) $a = -2, b = -6$ (d) $a = 2, b = 6$.

11) If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx + 8x - 9$ is negative of the other, then find the zeroes of $kx^2 + 3kx + 2$.

12) If α, β are zeroes of the quadratic polynomial $p(y) = 3y^2 - 6y + 4$ find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

13) Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

14) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, then prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

15) If one zero of $2x^2 - 3x + k$ is reciprocal to the other, then find the value of k

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X H.W-1 (Answers)

1) $x^2-\sqrt{2}x-12$ S: $-\sqrt{2}$ P: -12 → $-3\sqrt{2}, 2\sqrt{2}$

$=(x-3\sqrt{2})(x+2\sqrt{2})$

$\therefore x=3\sqrt{2}, -2\sqrt{2}$ (b)

2) $4x^2+5\sqrt{2}x-3$

$4x^2+6\sqrt{2}x-\sqrt{2}x-3$ S: $5\sqrt{2}$ P: -12 → $6\sqrt{2}$, $-1\sqrt{2}$

$2\sqrt{2}x(\sqrt{2}x+3)-(\sqrt{2}x+3)$

$=(2\sqrt{2}x-1)(\sqrt{2}x+3)$

$\therefore x=\frac{1}{2\sqrt{2}}, \frac{-3}{\sqrt{2}}$

$=\frac{\sqrt{2}}{4}, -\frac{3\sqrt{2}}{2}$ (c)

3) $\frac{1}{6}(6x^2+x-12)$ S: 1 P: -72 → $-8, 9$

$=\frac{1}{6}(6x^2-8x+9x-12)$

$=\frac{1}{6}(2x(3x-4)+3(3x-4))$

$=\frac{1}{6}(2x+3)(3x-4)$

$\therefore x=-\frac{3}{2}, \frac{4}{3}$ (b)

4) $7x^2-\frac{11}{3}x-\frac{2}{3}=\frac{1}{3}(21x^2-11x-2)$ S: -11 P: -42 → $-14, 3$

$=\frac{1}{3}(21x^2-14x+3x-2)=\frac{1}{3}(7x(3x-2)+(3x-2))$

$=\frac{1}{3}(7x+1)(3x-2)$

$\therefore x=-\frac{1}{7}, \frac{2}{3}$ (a)

5) $x^2-(\text{sum of zeroes})x+\text{product of zeroes}$

$=x^2-3x-10$ (c)

6) Sum of zeroes $=\frac{3^{\times 2}}{5_{\times 2}}-\frac{1^{\times 5}}{2_{\times 5}}=\frac{6-5}{10}=\frac{1}{10}$

product of zeroes $=\frac{3}{5}\times-\frac{1}{2}=-\frac{3}{10}$

$\therefore x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$= x^2 - \frac{1}{10}x - \frac{3}{10}$

$= \frac{1}{10}(10x^2 - x - 3)$ or $10x^2 - x - 3$ (d)

7) Sum of zeroes $= \cancel{-88}\ -\frac{b}{a} = -88$

product of zeroes $= \frac{c}{a} = 125$

This condition is satisfied only when both the zeroes are negative (b)

8) $\alpha + \beta = -\frac{b}{a} = -5$ (b)

9) $\alpha\beta = \frac{c}{a} = -\frac{9}{2}$ (c)

10) ~~when x =~~ Sum of zeroes $= -\frac{b}{A} = -(a+1)$

$\Rightarrow -2+3 = -a-1$

$\Rightarrow 1+1 = -a$

$\therefore a = -2$

Product of zeroes $= \frac{c}{A} = b$

$\Rightarrow -2 \times 3 = b$

$\therefore b = -6$ (c)

11) Let the zeroes be α and $-\alpha$

$f(x) = 4x^2 - 8x(k-1) - 9$

Sum of zeroes $= \alpha + (-\alpha) = -\frac{b}{a}$

$0 = \frac{\cancel{8}^{2}(k-1)}{\cancel{4}} = 2k-2$

$2 = 2k$

$k = 1$

Thus the polynomial is $x^2+3x+2 = (x+1)(x+2)$ $\quad$ $\overset{S}{3}\ \overset{P}{2}$, $1\ 2$

$\therefore$ The zeroes are -1 and -2.

12) Let $p(y) = 3y^2 - 6y + 4$ be of the form $ax^2 + bx + c$; where $a = 3, b = -6, c = 4$

$$\alpha + \beta = -\frac{b}{a} = \frac{6}{3} = 2$$

$$\alpha\beta = \frac{c}{a} = \frac{4}{3}$$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{4 - 2 \times \frac{4}{3}}{\frac{4}{3}} + \frac{\cancel{2}^{1} \times \cancel{2}^{1} \times 3}{\cancel{4}_{1}} + \cancel{3}^{1} \times \frac{4}{\cancel{3}_{1}}$$

$$= \frac{\frac{12-8}{3}}{\frac{4}{3}} + 3 + 4 = \frac{\frac{4}{3}}{\frac{4}{3}} + 7 = 1 + 7 = \underline{\underline{8}}$$

13) Let $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$ be of the form $ax^2 + bx + c$ where $a = 7, b = -\frac{11}{3}, c = -\frac{2}{3}$ and α, β be the zeroes.

$$7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$$

S P
−11 −42
−14, 3

$$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$$

$$= \frac{1}{3}[7y(3y-2) + (3y-2)]$$

$$= \frac{1}{3}(7y+1)(3y-2)$$

$\therefore y = \frac{-1}{7}, \frac{2}{3}$ are the zeroes of $p(y)$

Verification: Let $\alpha = -\frac{1}{7}, \beta = \frac{2}{3}$

Sum of zeroes $= \alpha+\beta = -\frac{1^{\times 3}}{7_{\times 3}} + \frac{2^{\times 7}}{3_{\times 7}} = \frac{-3+14}{21} = \frac{11}{21} = -\left(\frac{-11}{21}\right) = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

product of zeroes $= \alpha\beta = \frac{-1}{7} \times \frac{2}{3} = \frac{-2}{21} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence verified.

14) Let $f(x) = x^2 - px + q$ be of the form ax^2+bx+c; where

$a=1, b=-p, c=q$

$\alpha+\beta = -\frac{b}{a} = p$

$\alpha\beta = \frac{c}{a} = q$

Then, LHS, $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4+\beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$

$= \frac{[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$

$= \frac{(p^2-2q)^2 - 2q^2}{q^2}$

$= \frac{p^4 - 4p^2q + 4q^2 - 2q^2}{q^2}$

$= \frac{p^4 - 4p^2q + 2q^2}{q^2}$

$= \frac{p^4}{q^2} - \frac{4p^2q}{q^2} + \frac{2q^2}{q^2}$

$= \underline{\frac{p^4}{q^2} - \frac{4p^2}{q} + 2}$, RHS

15) Let the zeroes be α and $\frac{1}{\alpha}$ of $p(x) = 2x^2 - 3x + k$.

Then product of zeroes $= \alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$\Rightarrow 1 = \frac{k}{2}$

$\therefore \underline{\underline{k=2}}$