X Test-5

1) If $\frac{p}{q}$ is a rational number ($q \neq 0$) what is the condition of q so that the decimal representation of $\frac{p}{q}$ is terminating?

2) Check whether 4^n can end with digit 0 for any natural no. n

3) If two zeroes of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial.

4) Find the zeroes of $3x^2 + 5x - 2$ and verify the relationship between the zeroes and the coefficients

5) On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $(x-2)$ and $(-2x+4)$ respectively. Find $g(x)$.

6) Solve for x and y : $(a-b)x + (a+b)y = a^2 - 2ab - b^2$
by elimination method, $(a+b)(x+y) = a^2 + b^2$

7) Solve for x and y using cross multiplication method:

$$x + y = a + b$$
$$ax - by = a^2 - b^2$$

8) Solve for x and y : $37x + 41y = 70$

$$41x + 37y = 86$$

9) If α and β are the zeroes of $f(x) = x^2 - p(x+1) - c$, Show that $(\alpha+1)(\beta+1) = 1 - c$.

10) Draw the graphs of the following equations on the same graph paper: $2x + y = 2$; $2x + y = 6$.
Find the coordinates of the vertices of the trapezium formed by these lines. Also, find the area of the trapezium so formed.

X Test-5 (Answers)

1) If $\frac{p}{q}$ is a rational number, $q \neq 0$, the condition of q for the decimal representation to terminate is q must be of the form $2^m \times 5^n$; where m and n are non-negative integers.

2) $4^n = (2^2)^n = 2^{2n}$

We know that for a number to end with digit 0, the prime factorisation must contain 2 and 5 as factors. The prime factorisation of 4^n doesnot contain 5 as its factor. According to fundamental arithmetic theorem, there is no other prime factors for 4^n. Hence there is no natural number n for which 4^n ends with digit 0.

3) Let $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$.

Since $\sqrt{2}$ and $-\sqrt{2}$ are zeroes of $p(x)$,

$(x - \sqrt{2})$ and $(x + \sqrt{2})$ are its factors.

Also $x^2 - 2$ is a factor of $p(x)$.

On dividing $p(x)$ by $x^2 - 2$,

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & \cancel{2x^4} \mp 0x^3 - 4x^2 \quad (-)\,(-)\,(+) \\
\hline
 & -3x^3 + x^2 + 6x - 2 \\
 & \cancel{-3x^3} + 0x^2 + 6x \quad (+)\,(-)\,(-) \\
\hline
 & x^2 - 2 \\
 & \cancel{x^2} - 2 \quad (-)\,(+) \\
\hline
 & 0
\end{array}
$$

Thus quotient, $q(x) = 2x^2 - 3x + 1$

remainder, $r(x) = 0$

Using division algorithm,

$p(x) = (x^2 - 2)(2x^2 - 3x + 1) + 0$

$= (x^2 - 2)(2x^2 - x - 2x + 1)$

$= (x^2 - 2)[x(2x - 1) - (2x - 1)]$

$= (x^2 - 2)(x - 1)(2x - 1)$

S	P	
		-1
-3	2	-2

$\therefore$ Other zeroes of $p(x)$ are 1 and $\frac{1}{2}$ //

4) Let $p(x) = 3x^2 + 5x - 2$ be of the form $ax^2 + bx + c$; where $a = 3$, $b = 5$ and $c = -2$ and α, β be its zeroes.

$$p(x) = 3x^2 + 5x - 2$$
$$= 3x^2 + 6x - x - 2$$
$$= 3x(x+2) - 1(x+2)$$
$$= (3x-1)(x+2)$$

S P
5 −6 $<$ 6, −1

$\therefore$ The zeroes of $p(x)$ are $\frac{1}{3}$ and -2

Let $\alpha = \frac{1}{3}$ and $\beta = -2$

Then, $\alpha + \beta = \frac{1}{3} - 2 = -\frac{5}{3} = -\frac{b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\alpha\beta = \frac{1}{3} \times -2 = -\frac{2}{3} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence verified

5) Let $p(x) = x^3 - 3x^2 + x + 2$, $q(x) = x - 2$, $r(x) = -2x + 4$

Using division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$
$$x^3 - 3x^2 + x + 2 = g(x)(x-2) - 2x + 4$$
$$x^3 - 3x^2 + x + 2x + 2 - 4 = g(x)(x-2)$$
$$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

On dividing,

$$\begin{array}{r} x^2 - x + 1 \\ x - 2 \overline{\big) \cancel{x^3} - 3x^2 + 3x - 2} \\ \underset{(-)}{}x^3 \underset{(+)}{-} 2x^2 \\ \hline -\cancel{x^2} + 3x - 2. \\ \underset{(+)}{-}x^2 \underset{(-)}{+} 2x \\ \hline x - 2 \\ x - 2 \\ \hline 0 \end{array}$$

$\therefore g(x) = x^2 - x + 1$ //

6) $(a-b)x + (a+b)y = a^2 - 2ab - b^2 \longrightarrow (1)$

$\overset{(-)}{}(a+b)x \overset{(-)}{+} (a+b)y \overset{(-)}{=} (a^2 + b^2) \longrightarrow (2)$

$(1)-(2)$ $x(\cancel{a} - b - \cancel{a} - b) = \cancel{a^2} - 2ab - b^2 - \cancel{a^2} - b^2$

$\Rightarrow -2bx = -2ab - 2b^2$

$\Rightarrow \cancel{-2b}x = \cancel{-2b}(a+b)$

$\therefore \boxed{x = a+b}$

From eq: (1), $(a-b)(a+b) + (a+b)y = a^2 - 2ab - b^2$

$\Rightarrow \cancel{a^2 - b^2} + (a+b)y = \cancel{a^2} - 2ab \cancel{- b^2}$

$\Rightarrow \boxed{y = \dfrac{-2ab}{a+b}}$

7) $x + y - (a+b) = 0 \longrightarrow (1)$

$ax - by - (a^2 - b^2) = 0 \longrightarrow (2)$

$$\begin{matrix} 1 & 1 & -(a+b) \\ a & -b & -(a^2-b^2) \end{matrix}$$

$$\frac{x}{\begin{vmatrix} 1 & -(a+b) \\ -b & -(a^2-b^2) \end{vmatrix}} = \frac{y}{\begin{vmatrix} -(a+b) & 1 \\ -(a^2-b^2) & a \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & 1 \\ a & -b \end{vmatrix}}$$

$$\frac{x}{-(a^2-b^2) - b(a+b)} = \frac{y}{-a(a+b) + (a^2-b^2)} = \frac{1}{-b-a}$$

$$\frac{x}{-a^2 \cancel{+b^2} - ab \cancel{- b^2}} = \frac{y}{\cancel{-a^2} - ab \cancel{+a^2} - b^2} = \frac{1}{-(a+b)}$$

$$\frac{x}{-a(a+b)} = \frac{y}{-b(a+b)} = \frac{1}{-(a+b)}$$

$\therefore x = \dfrac{\cancel{-}a\cancel{(a+b)}}{\cancel{-}\cancel{(a+b)}} = \underline{\underline{a}}$

$y = \dfrac{\cancel{-}b\cancel{(a+b)}}{\cancel{-}\cancel{(a+b)}} = \underline{\underline{b}}$

8) $37x + 41y = 70 \longrightarrow (1)$

$41x + 37y = 86 \longrightarrow (2)$

$(1)+(2)$, $78x + 78y = 156$

$\div 78$, $x + y = 2 \longrightarrow (3)$

$(1)-(2)$, $-4x + 4y = -16$

$\div(-4)$, $x - y = 4 \longrightarrow (4)$

$(3)+(4)$, $2x = 6$

$\boxed{x = 3}$

From eq: (3), $y = 2 - 3$

$\boxed{y = -1}$

9) Let $p(x) = x^2 - p(x+1) - c = x^2 - px - p - c$ be of the form $ax^2 + bx + c$; where $a = 1$, $b = -p$, $c = -p - c$.

Then, $\alpha + \beta = -\frac{b}{a} = p$

$\alpha\beta = \frac{c}{a} = -p - c$

Thus, $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

$= \cancel{-p} - c \cancel{+p} + 1$

$= \underline{\underline{1 - c}}$

10) (graph)

b) $2x + y = 2$

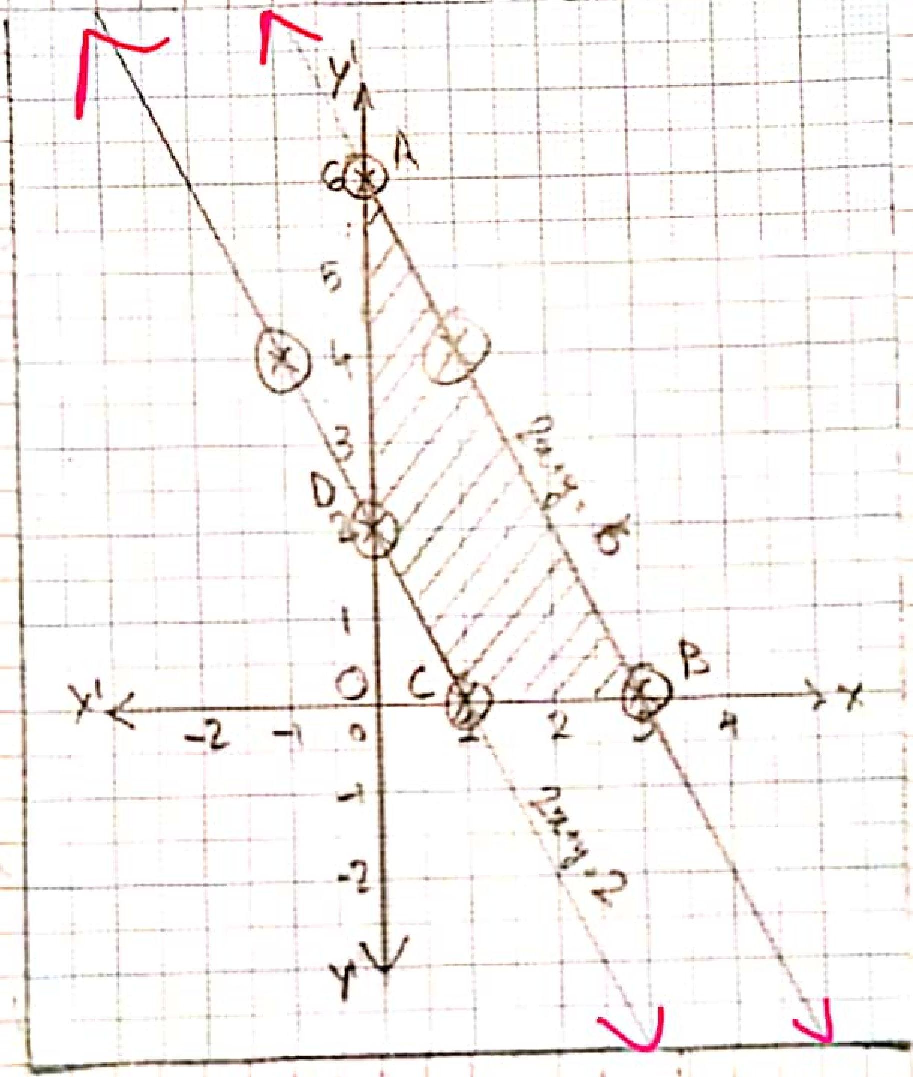
$x = \frac{2-y}{2}$

x	1	0	-1
y	0	2	4

$2x + y = 6$

$y = 6 - 2x$

x	0	3	1
y	6	0	4



The coordinates of the vertices of the trapezium formed is (0,6); (3,0); (1,0) and (0,2)

The area of this trapezium is

$\Rightarrow$ area of Δle BOA - area of Δle CDO

$= \left(\frac{1}{\cancel{2}} \times 3 \times \cancel{6}^{3}\right) - \left(\frac{1}{\cancel{2}} \times 1 \times \cancel{2}\right)$

$= 9 - 1 = \underline{8 \text{ units}^2}$