X

Test – 2

1) (1) If one of the zeroes of the Cubic polynomial x^3+ax^2+bx+c is -1, then the product of the other two zeroes is
(a) $b-a+1$ (b) $b-a-1$ (c) $a-b+1$ (d) $a-b-1$

2) (1) If a cubic polynomial with the sum of its zeroes, sum of the product and its zeroes taken two at a time and product of its zeroes as $2, -5$ and -11 respectively, then cubic polynomial is (a) x^3+7x-6 (b) x^3+7x+6
(c) x^3-7x-6 (d) x^3-7x+6

3) (1) If α and β are zeroes and the quadratic polynomial $p(s)=3s^2-6s+4$, then the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$ is
(a) 7 (b) 6 (c) 8 (d) 10

4) (1) If the square of difference of the zeroes of the quadratic polynomial $x^2+px+45$ is equal to 144, then value of p is
(a) ± 9 (b) ± 12 (c) ± 15 (d) ± 18

5) (1) If one zero of the cubic polynomial ax^3+bx^2+cx+d is zero, the product of then other two zeroes is
(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

6) (3) If α and β are the zeroes of $f(x)=x^2-px+q$, then prove that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$

7) (3) If one zero of the polynomial $(a^2+9)x^2+13x+6a$ is reciprocal of the other, then find the value of a.

8) (4) If the polynomial $6x^4+8x^3+17x^2+21x+7$ is divided by another polynomial $3x^2+4x+1$, then what will be the quotient and remainder?

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4) $p(x) = x^2 + px + 45$; $\alpha + \beta = -\frac{b}{a} = -p$; $\alpha\beta = \frac{c}{a} = 45$

$(\alpha - \beta)^2 = 144$

$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = 144$ $\quad [\because (a-b)^2 = (a+b)^2 - 4ab]$

$\Rightarrow (-p)^2 - 4 \times 45 = 144$

$\Rightarrow p^2 = 144 + 180$

$\Rightarrow p^2 = 324$

$p = \pm 18$ (d)

5) $p(x) = ax^3 + bx^2 + cx + d$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\Rightarrow 0 + 0 + \gamma\alpha = \frac{c}{a}$ $\quad$ [Let $\beta = 0$]

$\Rightarrow$ product of other two zeroes $= \frac{c}{a}$ (b)

6) $f(x) = x^2 - px + q$ be of the form $ax^2 + bx + c$; where $a = 1$, $b = -p$, $c = q$

$\alpha + \beta = -\frac{b}{a} = p$

$\alpha\beta = \frac{c}{a} = q$

$$\therefore \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2)^2 + (\beta^2)^2}{(\alpha\beta)^2} = \frac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{(\alpha\beta)^2}$$

$$= \frac{((\alpha+\beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{(p^2 - 2q)^2 - 2q^2}{q^2} = \frac{p^4 - 4p^2q + 4q^2 - 2q^2}{q^2}$$

$$= \frac{p^4 - 4p^2q + 2q^2}{q^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$$

7) Let $p(x) = (a^2+9)x^2 + 13x + 6a$ be in the form $Ax^2 + Bx + C$;
$A = a^2 + 9$, $B = 13$, $C = 6a$ and α and $\frac{1}{\alpha}$ be the zeroes.

Then, product of zeroes $= \alpha \times \frac{1}{\alpha} = \frac{c}{A}$

$$\Rightarrow 1 = \frac{6a}{a^2+9}$$

$$\Rightarrow a^2+9 = 6a$$

$$\Rightarrow a^2 - 6a + 9 = 0$$

$$\Rightarrow (a-3)^2 = 0$$

$$\Rightarrow a - 3 = 0$$

$$\therefore a = 3$$

8) Let $p(x) = 6x^4 + 8x^3 + 17x^2 + 21x + 7$ and $g(x) = 3x^2 + 4x + 1$.

On dividing $p(x)$ by $g(x)$,

$$\begin{array}{r|l} & 2x^2 + 5 \\ \hline 3x^2+4x+1 & \cancel{6x^4} + \cancel{8x^3} + 17x^2 + 21x + 7 \\ & \underline{\cancel{6x^4} + \cancel{8x^3} + 2x^2} \\ & \qquad 15x^2 + 21x + 7 \\ & \qquad \underline{\cancel{15x^2} + 20x + 5} \\ & \qquad\qquad x + 2 \end{array}$$

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Quotient $= q(x) = 2x^2 + 5$

Remainder $= r(x) = x + 2$.

X Test-2 (Solutions)

1) Let $p(x) = x^3 + ax^2 + bx + c$ be in the form $Ax^3 + Bx^2 + Cx + D$; where $A = 1$, $B = a$, $C = b$, $D = c$

Since -1 is a zero of $p(x)$, $p(-1) = 0$

$$\Rightarrow (-1)^3 + a(-1)^2 + b(-1) + c = 0$$
$$\Rightarrow -1 + a - b + c = 0$$
$$\Rightarrow c = 1 - a + b$$
$$\Rightarrow c = b - a + 1 \longrightarrow (1)$$

Product of zeroes $= \alpha\beta\gamma = -\frac{D}{A}$

$$\Rightarrow -1 \times \beta\gamma = -c$$
$$\Rightarrow \beta\gamma = c$$
$$\Rightarrow \beta\gamma = b - a + 1$$
$\Rightarrow$ product of other two zeroes $= b - a + 1$ (a)

2) $\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7$

$\alpha\beta\gamma = -6$

$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - 0x^2 - 7x - (-6)$

$= x^3 - 7x + 6$ (d)

3) $p(s) = 3s^2 - 6s + 4$ be in the form $ax^2 + bx + c$; $a = 3, b = -6, c = 4$

$\alpha + \beta = -\frac{b}{a} = \frac{6}{3} = 2$

$\alpha\beta = \frac{c}{a} = \frac{4}{3}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\beta + \alpha}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$

$= \frac{4 - 2 \times \frac{4}{3}}{\frac{4}{3}} + \cancel{2}^{1} \times \frac{\cancel{2}^{1}}{\cancel{4}} \times 3 + \cancel{3} \times \frac{4}{\cancel{3}}$

$\frac{4 - \frac{8}{3}}{\frac{4}{3}} + 3 + 4$

$= \frac{\cancel{\frac{4}{3}}^{1}}{\cancel{\frac{4}{3}}^{1}} + 7$

$= 1 + 7$

$= 8$ (c)