IX Test-6

1) Factorise : $\dfrac{1.03\times1.03\times1.03-0.03\times0.03\times0.03}{1.03\times1.03+1.03\times0.03+0.03\times0.03}$

2) Polynomial $3x^3-5x^2+kx-2$ and $-x^3-x^2+7x+k$ leave the same remainder when divided by $(x+2)$. Find the value at k.

3) Without actual division, Show that $f(x)=2x^4-6x^3+3x^2+3x-2$ is exactly divisible by x^2-3x+2.

4) If $x+4$ is a factor of the polynomial $x^3-x^2-14x+24$, find its other factors.

5) Expand : $\left(\frac{a}{4}-\frac{b}{2}+1\right)^2$

6) Expand : $\left(\frac{1}{3}x-\frac{2}{3}y\right)^3$

7) Factorize : x^4-y^4

8) Factorize : $p^3q^3+\frac{343}{729}$

9) Factorize : $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$

10) Without actually Calculating the Cubes, evaluate (i) $14^3+13^3-27^3$ (ii) $(102)^3$

IX test-6 (Answers)

1) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a = 1.03$; $b = 0.03$

$$\frac{(1.03)^3 - (0.03)^2}{(1.03)^2 + 1.03\times0.03 + (0.03)^2} = \frac{(1.03-0.03)\cancel{(1.03^2+1.03\times0.03+0.03^2)}}{\cancel{1.03^2+1.03\times0.03\times0.03^2}}$$

$= 1.03 - 0.03$

$= \underline{1}$

2) Let $p(x) = 3x^3 - 5x^2 + kx - 2$ and $f(x) = -x^3 - x^2 + 7x + k$

Since $p(x)$ and $f(x)$ leaves same remainder when divided by $(x+2)$,

$p(-2) = f(-2)$

$\Rightarrow 3(-2)^3 - 5(-2)^2 + k\times-2 - 2 = -(-2)^3 - (-2)^2 + 7(-2) + k$

$\Rightarrow -24 - 20 - 2k - 2 = 8 - 4 - 14 + k$

$\Rightarrow -46 - 2k = -10 + k$

$\Rightarrow -36 = 3k$

$\therefore k = \frac{-36}{3} = \underline{\underline{-12}}$

3) $x^2 - 3x + 2 = (x-1)(x-2)$

S P
-3 2 < -1, -2

$f(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$

$f(1) = 2 - 6 + 3 + 3 - 2 = 8 - 8 = 0$ //

$f(2) = 2\times2^4 - 6\times2^3 + 3\times2^2 + 3\times2 - 2$

$= 32 - 48 + 12 + 6 - 2 = 50 - 50 = 0$ //

Thus $(x-1)$ and $(x-2)$ are the factors of $f(x)$.

Then, $(x-1)(x-2) = x^2 - 3x + 2$ is also a factor

Hence $f(x)$ is exactly divisible by $x^2 - 3x + 2$.

4) Let $p(x) = x^3 - x^2 - 14x + 24$.

On dividing $p(x)$ by $(x+4)$,

quotient $= x^2 - 5x + 6$

Using division algorithm,

$p(x) = (x+4)(x^2 - 5x + 6)$

$= (x+4)(x-3)(x-2)$

S P
-5 6
-3, -2

$$\begin{array}{r|l} & x^2 - 5x + 6 \\ \hline x+4 & x^3 - x^2 - 14x + 24 \\ & -(x^3 + 4x^2) \\ \hline & -5x^2 - 14x + 24 \\ & -(-5x^2 - 20x) \\ \hline & 6x + 24 \\ & -(6x + 24) \\ \hline & 0 \end{array}$$

5) $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$\left(\frac{a}{4}-\frac{b}{2}+1\right)^2 = \left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(1)^2+\cancel{2}^{1}\times\frac{a}{4}\times\frac{-b}{\cancel{2}_1}+\cancel{2}^{1}\times\frac{-b}{\cancel{2}_1}\times 1+$

$\cancel{2}^{1}\times 1\times\frac{a}{\cancel{4}_2}$

$= \frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}$

6) $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$\left(\frac{1}{3}x-\frac{2}{3}y\right)^3 = \left(\frac{1}{3}x\right)^3-3\left(\frac{1}{3}x\right)^2\times\frac{2}{3}y+\cancel{3}\times\frac{1}{\cancel{3}_1}x\times\left(\frac{2}{3}y\right)^2-\left(\frac{2}{3}y\right)^3$

$= \frac{x^3}{27}-\cancel{3}^{1}\times\frac{x^2}{9}\times\frac{2}{\cancel{3}_1}y+\frac{4xy^2}{9}-\frac{8y^3}{27}$

$= \frac{x^3}{27}-\frac{2x^2y}{9}+\frac{4xy^2}{9}-\frac{8y^3}{27}$

7) $x^4-y^4 = (x^2)^2-(y^2)^2$

$= (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x+y)(x-y)$

8) $a^3+b^3 = (a+b)(a^2-ab+b^2)$

$p^3q^3+\frac{343}{729} = (pq)^3+\left(\frac{7}{9}\right)^3$

$= \left(pq+\frac{7}{9}\right)\left((pq)^2-pq\times\frac{7}{9}+\left(\frac{7}{9}\right)^2\right)$

$= \left(pq+\frac{7}{9}\right)\left(p^2q^2-\frac{7pq}{9}+\frac{49}{81}\right)$

9) $\left(x^2+\frac{1}{x^2}+2\right)-2\left(x+\frac{1}{x}\right) = \left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)$

$$= \left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$$

$$= \left(x+\frac{1}{x}\right)\left((\sqrt{x})^2+\frac{1}{(\sqrt{x})^2}-2\right)$$

$$= \left(x+\frac{1}{x}\right)\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2$$

$$= \left(x+\frac{1}{x}\right)\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$$

10) (i) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Checking :- $a+b+c = 14+13+(-27) = 27-27=0$

$\therefore\ 14^3+13^3+(-27)^3 = 3\times14\times13\times-27$

$= -\underline{\underline{14742}}$

(ii) $(102)^3 = (100+2)^3$

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$= 100^3+2^3+3\times100\times2\,(100+2)$

$= 1000000+8+600\times102$

$= 1000008+61200$

$= \underline{\underline{1061208}}$