IX H.W-7

1) If $x^2+\frac{1}{x^2}=66$, find the value of $x-\frac{1}{x}$

2) If $x^2+\frac{1}{x^2}=79$, find the value of $x+\frac{1}{x}$

3) If $3x-7y=10$ and $xy=-1$, find the value of $9x^2+49y^2$

4) If $2x+3y=8$ and $xy=2$, find the value of $4x^2+9y^2$

5) If $9x^2+25y^2=181$ and $xy=-6$, find the value of $3x+5y$

6) If $x-\frac{1}{x}=-1$, find the value of $x^2+\frac{1}{x^2}$

7) Evaluate (i) $(0.98)^2$ (ii) 991×1009

8) If $a+b+c=9$ and $ab+bc+ca=23$, find the value of $a^2+b^2+c^2$

9) If $a^2+b^2+c^2=16$ and $ab+bc+ca=10$, find the value of $a+b+c$.

10) Simplify: $(2x-5y)^3-(2x+5y)^3$

11) If $x^2+\frac{1}{x^2}=7$, find the value of $x^3+\frac{1}{x^3}$

12) If $3x-2y=11$ and $xy=12$, find the value of $27x^3-8y^3$

13) If $x-\frac{1}{x}=7$, find the value of $x^3-\frac{1}{x^3}$

14) If $a+b=10$ and $ab=21$, find the value of a^3+b^3

15) Evaluate: $48^3-30^3-18^3$

16) If $a+b+c=9$ and $a^2+b^2+c^2=35$, find $a^3+b^3+c^3-3abc$

17) Factorise: $2(x+y)^2-9(x+y)-5$

18) Simplify: $\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$

19) Factorise: $\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

20) Find the value of $x^3-8y^3-36xy-216$ when $x=2y+6$

21) Factorise: $3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$

22) Factorise: $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

IX HW-7 (Answers)

1) $(a-b)^2 = a^2+b^2-2ab$

$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$

$= 66-2 = 64$

$\therefore x-\frac{1}{x} = \sqrt{64} = \underline{8}$

2) $(a+b)^2 = a^2+b^2+2ab$

$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$= 79+2 = 81$

$\therefore x+\frac{1}{x} = \sqrt{81} = \underline{9}$

3) $a^2+b^2 = (a-b)^2+2ab$

$9x^2+49y^2 = (3x)^2+(7y)^2$

$= (3x-7y)^2+2\times3x\times7y$

$= (3x-7y)^2+42xy$

$= 10^2+42\times(-1)$

$= 100-42 = \underline{58}$

4) $a^2+b^2 = (a+b)^2-2ab$

$4x^2+9y^2 = (2x)^2+(3y)^2$

$= (2x+3y)^2-2\times2x\times3y$

$= 8^2-12\times2$

$= 64-24 = \underline{40}$

5) $(a+b)^2 = a^2+b^2+2ab$

$(3x+5y)^2 = 9x^2+25y^2+2\times3x\times5y$

$= 9x^2+25y^2+30xy$

$= 181+30\times-6$

$= 181-180 = \underline{1}$

6) $(a-b)^2 = a^2+b^2-2ab$

$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2$

$(-1)^2 = x^2+\frac{1}{x^2}-2$

$1+2 = x^2+\frac{1}{x^2}$

$\therefore x^2 + \frac{1}{x^2} = \underline{\underline{3}}$

7) (i) $(0.98)^2 = (1 - 0.02)^2$

$(a-b)^2 = a^2 + b^2 - 2ab$

$= 1^2 + (0.02)^2 - 2 \times 1 \times 0.02$

$= 1 + 0.0004 - 0.04$

$= \underline{0.9604}$

(ii) $991 \times 1009 = (1000 - 9)(1000 + 9)$

$(a+b)(a-b) = a^2 - b^2$

$= (1000)^2 - (9)^2$

$= 1000000 - 81 = \underline{\underline{999919}}$

8) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$9^2 = a^2 + b^2 + c^2 + 2 \times 23$

$81 - 46 = a^2 + b^2 + c^2$

$\therefore a^2 + b^2 + c^2 = \underline{\underline{35}}$

9) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= 16 + 2 \times 10$

$= 16 + 20$

$= 36$

$\therefore (a+b+c) = \sqrt{36} = \underline{\underline{6}}$

10) $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$; $a = 2x - 5y$; $b = 2x + 5y$

$(2x - 5y)^3 - (2x + 5y)^3 = (\cancel{2x} - 5y - \cancel{2x} - 5y)^3 + 3(2x - 5y)(2x + 5y)$

$(\cancel{2x} - 5y - \cancel{2x} - 5y)$

$= (-10y)^3 + 3(4x^2 - 25y^2) \times -10y$

$= -1000y^3 - 30y(4x^2 - 25y^2)$

$= -1000y^3 - 120x^2y + 750y^3$

$= \underline{\underline{-250y^3 - 120x^2y}}$

11) $(a+b)^2 = a^2 + b^2 + 2ab$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 7 + 2 = 9$

$x + \frac{1}{x} = \sqrt{9} = 3 \longrightarrow (1)$

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$x^3+\frac{1}{x^3} = (x+\frac{1}{x})(x^2+\frac{1}{x^2}-1)$

$= 3(7-1) = 3\times 6 = \underline{18}$

12) $(a-b)^2 = a^2+b^2-2ab$

$(3x-2y)^2 = (3x)^2+(2y)^2-2\times 3x\times 2y$

$= 9x^2+4y^2-12xy$

$11^2 = 9x^2+4y^2-12\times 12$

$121+144 = 9x^2+4y^2$

$9x^2+4y^2 = 265$

$a^3-b^3 = (a-b)(a^2+b^2+ab)$

$27x^3-8y^3 = (3x)^3-(2y)^3 = (3x-2y)(9x^2+4y^2+6xy)$

$= 11(265+6\times 12)$

$= 11(265+72) = 11\times 337$

$= \underline{3707}$

(Or)

$a^3-b^3 = (a-b)^3+3ab(a-b)$

$27x^3-8y^3 = (3x)^3-(2y)^3 = (3x-2y)^3+3\times 3x\times 2y(3x-2y)$

$= (3x-2y)^3+18xy(3x-2y)$

$= 11^3+18\times 12\times 11$

$= 1331+2376 = \underline{3707}$

13) $a^3-b^3 = (a-b)^3+3ab(a-b)$

$x^3-\frac{1}{x^3} = (x-\frac{1}{x})^3+3x\times\frac{1}{x}(x-\frac{1}{x})$

$= 7^3+3\times 7 = 343+21 = \underline{364}$

14) $a^3+b^3 = (a+b)^3-3ab(a+b)$

$a^3+b^3 = 10^3-3\times 21\times 10$

$= 1000-630 = \underline{370}$

15) $a^3+b^3+c^3 = 3abc$, if $a+b+c=0$

checking :- $48+(-30)+(-18) = 48-48 = 0$

$\therefore (48)^3+(-30)^3+(-18)^3 = 3\times 48\times -30\times -18$

$= \underline{77760}$

16) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$a^3+b^3+c^3-3abc = (a+b+c)[a^2+b^2+c^2-(ab+bc+ca)]$

$= 9[35-23]$ [from eq: (1)]

$= 9 \times 12$

$= \underline{108}$

$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$9^2 = 35+2(ab+bc+ca)$

$81-35 = 2(ab+bc+ca)$

$ab+bc+ca = \frac{46}{2} = 23 \rightarrow (1)$

17) Put $x+y = a$

Then, $2a^2-9a-5$

$= 2a^2+a-10a-5$

$= a(2a+1)-5(2a+1)$

$= (a-5)(2a+1)$

$= (x+y-5)(2x+2y+1)$

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-9 -10 < -10

18) $a^3+b^3 = (a+b)(a^2+b^2-ab)$

$$\frac{(173)^3+(127)^3}{(173)^2-173\times127+(127)^2} = \frac{(173+127)(173^2-173\times127+127^2)}{173^2-173\times127+127^2}$$

$= 173+127 = \underline{300}$

19) $\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

$= \left(\frac{2}{3}x\right)^3+(1)^3+3\times\left(\frac{2}{3}x\right)^2\times1+3\times\frac{2}{3}x\times1^2$

$= \left(\frac{2}{3}x+1\right)^3$ $\quad [a^3+b^3+3a^2b+3ab^2 = (a+b)^3]$

$= \left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

20) Given, $x = 2y+6$

$\Rightarrow x-2y-6 = 0 \rightarrow (1)$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$x^3-8y^3-36xy-216$

$= (x)^3+(-2y)^3+(-6)^3-3\times x\times-2y\times-6$

$= (x-2y-6)(x^2+(-2y)^2+(-6)^2-x\times-2y-(-2y)\times-6-x\times(-6)$

$= 0\times(x^2+4y^2+36+2xy-12y+6x)$

$= 0$

21) $3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$

$= (\sqrt{3}a)^3+(-b)^3+(\sqrt{5}c)^3-3\times\sqrt{3}a\times-b\times\sqrt{5}c$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= (\sqrt{3}a - b + -\sqrt{5}c)\left((\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - \sqrt{3}a \times -b - (-b) \times -\sqrt{5}c - (-\sqrt{5}c) \times \sqrt{3}a\right)$

$= (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$

22) If $a+b+c=0$, $a^3+b^3+c^3 = 3abc$

checking: $p(q-r) + q(r-p) + r(p-q)$

$= pq - pr + qr - pq + rp - rq = 0$

$\therefore (p(q-r))^3 + (q(r-p))^3 + (r(p-q))^3$

$= 3p(q-r) \times q(r-p) \times r(p-q)$

$= 3pqr(q-r)(r-p)(p-q)$