IX Homework - 6

1) If $x - 2y = 11$ and $xy = 8$, find the value of $x^3 - 8y^3$
2) Factorise : $a^7 + ab^6$
3) Multiply $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ by $3x - 5y - 4$ using suitable identity
4) Factorise : $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$.
5) If $8x^3 + 27y^3 = 730$ and $2x^2y + 3xy^2 = 15$, then evaluate $2x + 3y$.
6) Factorise : $27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}$
7) If $a + b + c = 12$, $a^2 + b^2 + c^2 = 90$, find the value of $a^3 + b^3 + c^3 - 3abc$.
8) If $a^2 + b^2 + c^2 = 250$ and $ab + bc + ac = 3$, find the value of $a + b + c$.
9) Simplify: $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$
10) Factorise : $2x^3 - 3x^2 - 17x + 30$
11) Factorise : $x^8 - y^8$
12) If $x + \frac{1}{x} = 7$, find the value of $x^3 + \frac{1}{x^3}$
13) If $x - \frac{1}{x} = 3$, then find the value of $x^3 - \frac{1}{x^3}$
14) Using suitable identity evaluate : $(-32)^3 + (18)^3 + (14)^3$
15) Without finding cubes, factorise and find the value of $\left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{7}{12}\right)^3$

IX Homework-6 (Answers)

1) $a^3-b^3=(a-b)^3+3ab(a-b)$

$x^3-8y^3=(x)^3-(2y)^3$

$=(x-2y)^3+3\times x\times 2y\,(x-2y)$

$=(x-2y)^3+6xy\,(x-2y)$

$=11^3+6\times 8\times 11$

$=1331+528=\underline{1859}$

2) $a^7+ab^6=a(a^6+b^6)$

$=a((a^2)^3+(b^2)^3)$ $\quad [x^3+y^3=(x+y)(x^2-xy+y^2)]$

$=a(a^2+b^2)(a^4-a^2b^2+b^4)$

3) $(9x^2+25y^2+15xy+12x-20y+16)(3x-5y-4)$

$=(3x)^2+(-5y)^2+(-4)^2-3x\times-5y-(-5y)\times(-4)-(3x)(-4)$

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=a^3+b^3+c^3-3abc$

$=(3x)^3+(-5y)^3+(-4)^3-3\times3x\times-5y\times-4$

$=27x^3-125y^3-64-180xy$

4) $2\sqrt{2}a^3+8b^3-27c^3+18\sqrt{2}abc$

$=(\sqrt{2}a)^3+(2b)^3+(-3c)^3-3\times\sqrt{2}a\times2b\times-3c$

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=(\sqrt{2}a+2b-3c)((\sqrt{2}a)^2+(2b)^2+(-3c)^2-\sqrt{2}a\times2b-2b\times-3c-(-3c)\times\sqrt{2}a$

$=(\sqrt{2}a+2b-3c)(2a^2+4b^2+9c^2-2\sqrt{2}ab+6bc+3\sqrt{2}ac$

5) $(a+b)^3=a^3+b^3+3ab(a+b)$

$(2x+3y)^3=(2x)^3+(3y)^3+3\times2x\times3y(2x+3y)$

$=8x^3+27y^3+2x^2y\times18+3xy^2\times18$

$=730+18(2x^2y+3xy^2)$

$=730+18\times15=730+270=\underline{1000}$

$\therefore 2x+3y=\sqrt[3]{1000}=10$//

6) $27a^3+\frac{1}{64b^3}+\frac{27a^2}{4b}+\frac{9a}{16b^2}=(3a)^3+\left(\frac{1}{4b}\right)^3+3\times(3a)^2\times\frac{1}{4b}+3\times3a\times\left(\frac{1}{4b}\right)^2$

$a^3+b^3+3a^2b+3ab^2=(a+b)^3$

$=\left(3a+\frac{1}{4b}\right)^3=\left(3a+\frac{1}{4b}\right)\left(3a+\frac{1}{4b}\right)\left(3a+\frac{1}{4b}\right)$

7) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= 12(90-(ab+bc+ca))$

$= 12(90-27)$

$= 12 \times 63$

$= \underline{\underline{756}}$

$(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$

$\Rightarrow 12^2 = 90 + 2(ab+bc+ca)$

$\therefore ab+bc+ca = \frac{144-90}{2}$

$= \frac{54}{2} = \underline{\underline{27}}$

8) $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$= 250 + 2 \times 3$

$= 250 + 6$

$= 256$

$\therefore (a+b+c) = \sqrt{256} = \underline{\underline{16}}$

9) If $x+y+z=0$, $x^3+y^3+z^3 = 3xyz$

Checking:- $a^2-b^2+b^2-c^2+c^2-a^2 = 0$

$a-b+b-c+c-a = 0$

$\therefore \frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3} = \frac{3(a^2-b^2)(b^2-c^2)(c^2-a^2)}{3(a-b)(b-c)(c-a)}$

$= \frac{(a+b)(a-b)(b+c)(b-c)(c+a)(c-a)}{(a-b)(b-c)(c-a)}$

$= (a+b)(b+c)(c+a)$

10) Let $p(x) = 2x^3-3x^2-17x+30$

factors of 60 are $\pm 1, \pm 2, \pm 3$ etc

$p(1) = 2-3-17+30 = 32-20 = 12 \neq 0$

$p(-1) = 2(-1)^3-3(-1)^2-17(-1)+30 = -2-3+17+30 = 42 \neq 0$

$p(2) = 2(2)^3-3(2)^2-17\times 2+30 = 16-12-34+30 = 46-46$

$= 0$

Thus $(x-2)$ is a factor of $p(x)$

On dividing $p(x)$ by $(x-2)$,

Quotient, $q(x) = 2x^2+x-15$

Using division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$= (x-2)(2x^2+x-15) + 0$

$= (x-2)(2x^2+6x-5x-15)$

$= (x-2)[2x(x+3)-5(x+3)]$

$= (x-2)(2x-5)(x+3)$

$$x-2 \;\overline{)\; 2x^3 - 3x^2 - 17x + 30 \;(\; 2x^2+x-15}$$

$2x^3 - 4x^2$

$x^2 - 17x + 30$

$x^2 - 2x$

$-15x + 30$

$-15x + 30$

0

S	P
1	-30
	$6, -5$

11) $x^8 - y^8 = (x^4)^2 - (y^4)^2$ $\quad [a^2-b^2 = (a+b)(a-b)]$

$= (x^4+y^4)(x^4-y^4)$

$= (x^4+y^4)((x^2)^2-(y^2)^2)$

$= (x^4+y^4)(x^2+y^2)(x^2-y^2)$

$= (x^4+y^4)(x^2+y^2)(x+y)(x-y)$

12) $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3 \times x \times \frac{1}{x}\left(x+\frac{1}{x}\right)$

$= 7^3 - 3 \times 7 = 343 - 21 = \underline{322}$

13) $a^3-b^3 = (a-b)^3 + 3ab(a-b)$

$x^3 - \frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3 \times x \times \frac{1}{x}\left(x-\frac{1}{x}\right)$

$= 3^3 + 3 \times 3 = 27 + 9 = \underline{36}$

14) If $a+b+c=0$, $a^3+b^3+c^3 = 3abc$

Checking: $(-32)+(18)+(14) = -32+32 = 0$

$\therefore (-32)^3 + (18)^3 + (14)^3 = 3 \times -32 \times 18 \times 14 = \underline{-24192}$

15) If $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$

checking: $\frac{1}{4\times3} + \frac{1}{3\times4} - \frac{7}{12} = \frac{3+4-7}{12} = \frac{7-7}{12} = 0$

$\therefore \left(\frac{1}{4}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{7}{12}\right)^3 = 3 \times \frac{1}{4} \times \frac{1}{3} \times -\frac{7}{12} = \underline{-\frac{7}{48}}$