IX Homework-5

1) The polynomials $ax^3 - 3x^2 + 4$ and $3x^2 - 5x + a$ when divided by $(x-2)$ leave the remainders p and q respectively. If $p - 2q = a$, find the value of a. $a=2$

2) Without actual division, prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

3) The polynomials $p(x) = ax^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + a$ leave the same remainder when divided by $(x-3)$, find the value of a and the remainder when $p(x)$ is divided by $(x-2)$. $a = -1$ $p(2) = 10$

4) If $p(x) = x^3 - ax^2 + bx + 3$ leaves a remainder -19 when divided by $(x+2)$ and a remainder 17 when divided by $(x-2)$, P.T $a+b = 6$.

5) Find the values of m and n so that the polynomial $f(x) = x^3 - 6x^2 + mx - n$ is exactly divisible by $(x-1)$ as well as $(x-2)$. $m = 11, n = 6$

6) If $x = -\frac{1}{3}$ is a zero of the polynomial $p(x) = 27a^3 - ax^2 - x + 3$, then find the value of a. $a = 21$

7) If the polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 8$ is divided by $(x-2)$, it leaves a remainder 10. Find the value of a. $a = 5$

8) Find the values of p and q, if the polynomial $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by the polynomial $x^2 - 1$. $p = 3, q = -3$

9) If $x^3 + ax^2 + bx + 6$ has $(x-2)$ as a factor and leaves a remainder 3 when divided by $(x-3)$, find the values of a and b. $a = -3$ $b = -1$

10) Find the remainder when $3x^3 - 4x^2 + 7x - 5$ is divided by $(x-3)$ and $(x+3)$. $61, -143$

11) Check whether $(p+1)$ is a factor of $(p^{100} - 1)$ and $(p^{101} + 1)$.

12) Find the remainder obtained on dividing $2x^4 - 3x^3 - 5x^2 + x + 1$ by $(x - \frac{1}{2})$. 0

13) Factorise by splitting the middle term: $9(x-2y)^2 - 4(x-2y) - 13$

14) Prove that $\dfrac{1}{1 + x^{a-b}} + \dfrac{1}{1 + x^{b-a}} = 1$

15) If $a = \dfrac{4}{3 - \sqrt{5}}$. Find the value of $a + \dfrac{4}{a}$. 8

16) If $a = \dfrac{3 + \sqrt{7}}{2}$, then find the value of $a^2 + \dfrac{1}{a^2}$. $\dfrac{40 - 9\sqrt{7}}{2}$

1) Let $p_1(x) = ax^3 - 3x^2 + 4$ and $p_2(x) = 3x^2 - 5x + a$.

when $p_1(x)$ is divided by $(x-2)$, $p_1(2) = p$

$\Rightarrow 8a - 12 + 4 = p$

$\Rightarrow 8a - 8 = p \longrightarrow (1)$

When $p_2(x)$ is divided by $(x-2)$, $p_2(2) = q$

$\Rightarrow 12 - 10 + a = q$

$\Rightarrow 2 + a = q \longrightarrow (2)$

Given, $p - 2q = a$

$\Rightarrow 8a - 8 - 2(2+a) = a$

$\Rightarrow 8a - 8 - 4 - 2a = a$

$\Rightarrow 6a - 12 = a$

$\Rightarrow 5a = 12$

$\therefore a = \dfrac{12}{5}$

2) $x^2 - 3x + 2 = (x-1)(x-2)$

Let $p(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$.

$p(1) = 2 - 6 + 3 + 3 - 2 = 8 - 8 = 0$

$p(2) = 32 - 48 + 12 + 6 - 2 = 50 - 50 = 0$

Thus $p(x)$ is divisible by $(x-1)$ and $(x-2)$

i.e, $p(x)$ is exactly divisible by $(x-1)(x-2) = x^2 - 3x + 2$.

3) When $p(x)$ is divided by $(x-3)$, remainder $= p(3)$

When $q(x)$ is divided by $(x-3)$, remainder $= q(3)$

Thus $p(3) = q(3)$

$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$

$\Rightarrow 27a + 41 = 15 + a$

$\Rightarrow 26a = -26$

$a = \dfrac{-26}{26} = -1$

Thus $p(x) = -x^3 + 4x^2 + 3x - 4$

When $p(x)$ is divided by $(x-2)$, remainder $= p(2)$

$= -8 + 16 + 6 - 4 = -12 + 22 = 10$

4) $p(-2) = -19$

$\Rightarrow -8 - 4a - 2b + 3 = -19$

$\Rightarrow -4a - 2b = -14$

$\Rightarrow -2a - b = -7 \longrightarrow (1)$

$p(2) = 17$

$\Rightarrow 8 - 4a + 2b + 3 = 17$

$\Rightarrow -4a + 2b = 6$

$\Rightarrow -2a + b = 3 \longrightarrow (2)$

$(1) + (2), \quad -4a = -4$

$\qquad a = \underline{1}$

From eq: (2), $b = 3 + 2a = 3 + 2 = \underline{5}$

$\therefore a + b = 1 + 5 = \underline{\underline{6}}$

5) $f(1) = 0$

$\Rightarrow 1 - 6 + m - n = 0$

$\Rightarrow m - n = 5 \longrightarrow (1)$

$f(2) = 0$

$\Rightarrow 8 - 24 + 2m - n = 0$

$\Rightarrow 2m - n = +16 \longrightarrow (2)$

$(1) - (2), \quad -m = -11$

$\qquad m = \underline{11}$

From eq: (1), $11 - n = 5$

$\qquad -n = -6$

$\qquad n = \underline{6}$

(or)

$-2a - b = -7$

$-b = -7 + 2a$

$b = 7 - 2a \longrightarrow (1)$

$P(2) = 17$

$\Rightarrow -2a + b = 3$

$\Rightarrow -2a + 7 - 2a = 3 \quad \{\text{from eq}^{n}(1)\}$

$\Rightarrow -4a = -4$

$\qquad a = 1$

From eq: (1), $b = 7 - 2 = \underline{5}$

$\therefore a + b = 6$

(or)

$f(1) = 0$

$\Rightarrow m - n = 5$

$\Rightarrow m = 5 + n \longrightarrow (1)$

$f(2) = 0$

$\Rightarrow 2m - n = 16$

$\Rightarrow 2(5 + n) - n = 16 \quad [\text{from eq}^{n}(1)]$

$\Rightarrow 10 + 2n - n = 16$

$\Rightarrow n = \underline{6}$

From eq: (1), $m = 5 + 6 = \underline{11}$

6) Since $-\frac{1}{3}$ is a zero of $p(x)$, $p\left(-\frac{1}{3}\right) = 0$

$\Rightarrow \dfrac{-27}{27} - \dfrac{a}{9} + \dfrac{1}{3} + 3 = 0$

$= -1 - \dfrac{a}{9} + \dfrac{1}{3} + 3 = 0$

$= 2 + \dfrac{1}{3} - \dfrac{a}{9} = 0$

$= \dfrac{6+1}{3} - \dfrac{a}{9} = 0$

$\Rightarrow -\dfrac{a}{9} = \dfrac{-7}{3}$

$a = \dfrac{7 \times \cancel{9}^{3}}{\cancel{3}}$

$a = 21$

7) when $p(x)$ is divided by $(x-2)$, remainder = 10

$$p(2) = 10$$
$$\Rightarrow 16 - 16 + 12 - 2a + 8 = 10$$
$$\Rightarrow 20 - 2a = 10$$
$$\Rightarrow -2a = -10$$
$$\underline{a = 5}$$

8) $x^2 - 1 = (x+1)(x-1)$

Let $p(x) = x^4 + px^3 + 2x^2 - 3x + q$

$$p(-1) = 0$$
$$\Rightarrow 1 - p + 2 + 3 + q = 0$$
$$\Rightarrow -p + q = -6 \longrightarrow (1)$$

$$p(1) = 0$$
$$\Rightarrow 1 + p + 2 - 3 + q = 0$$
$$\Rightarrow p + q = 0 \longrightarrow (2)$$

$(1) + (2), 2q = -6$ | From eq : (2),
$\qquad q = -3 /\!/$ | $\qquad p - 3 = 0$
$\qquad\qquad\qquad$ | $\qquad\quad p = 3 /\!/$

9) Let $p(x) = x^3 + ax^2 + bx + 6$

when $p(x)$ is divided by $(x-2)$, $p(2) = 0$
$$\Rightarrow 8 + 4a + 2b + 6 = 6$$
$$\Rightarrow 4a + 2b = -14$$
$$\Rightarrow 2a + b = -7 \longrightarrow (1)$$

when $p(x)$ is divided by $(x-3)$, remainder = 3
$$\Rightarrow p(3) = 3$$
$$\Rightarrow 27 + 9a + 3b + 6 = 3$$
$$\Rightarrow 9a + 3b = -30$$
$$\Rightarrow 3a + b = -10 \longrightarrow (2)$$

$(1) - (2), -a = 3$ | From eq : (1), $-6 + b = -7$
$\qquad a = -3 /\!/$ | $\qquad\qquad\qquad b = -1 /\!/$

10) Let $p(x) = 3x^3 - 4x^2 + 7x - 5$

$p(3) = 3 \times 27 - 4 \times 9 + 21 - 5 = 81 - 36 + 21 - 5 = 102 - 41 = \underline{61}$

$p(-3) = 3 \times -27 - 4 \times 9 - 21 - 5 = -81 - 36 - 21 - 5 = \underline{-143}$

11) Let $p(x) = p^{100} - 1$

$p(-1) = (-1)^{100} - 1 = 1 - 1 = 0$ //

Let $f(x) = p^{101} + 1$

$f(-1) = (-1)^{101} + 1 = -1 + 1 = 0$ //

Yes, $(p+1)$ is a factor of $p(x)$ and $f(x)$.

12) Let $p(x) = 2x^4 - 3x^3 - 5x^2 + x + 1$.

$$p\left(\tfrac{1}{2}\right) = \frac{2^{1}}{\cancel{16}\,8} - \frac{3}{8} - \frac{5}{4} + \frac{1}{2} + 1$$

$$= \frac{-2^{1}}{\cancel{8}\,4} - \frac{5}{4} + \frac{1}{2} + 1$$

$$= -\frac{\cancel{6}^{3}}{\cancel{4}\,2} + \frac{1}{2} + 1$$

$$= -\frac{2}{2} + 1 = -1 + 1 = 0 \text{ //}$$

13) Let $x - 2y = a$

$9a^2 - 4a - 13$

$= 9a^2 + 9a - 13a - 13$

$= 9a(a+1) - 13(a+1)$

$= (9a - 13)(a+1)$

$= [9(x-2y) - 13][x - 2y + 1]$

$= (9x - 18y - 13)(x - 2y + 1)$ //

$$\begin{array}{c}
S \quad\quad P \\
-4 \quad -117 \,\big\langle{\,9 \atop -13}
\end{array}$$

$$\begin{array}{r} 3\,\lfloor 117 \\ 3\,\lfloor 39 \\ 13 \end{array}$$

14) LHS, $\dfrac{1}{1 + \dfrac{x^a}{x^b}} + \dfrac{1}{1 + \dfrac{x^b}{x^a}} = \dfrac{1}{\dfrac{x^b + x^a}{x^b}} + \dfrac{1}{\dfrac{x^a + x^b}{x^a}}$

$= \dfrac{x^b}{x^b + x^a} + \dfrac{x^a}{x^a + x^b} = \dfrac{x^b + x^a}{x^b + x^a} = 1, \text{RHS}$

15) $a = \dfrac{4}{3 - \sqrt5} = \dfrac{4(3 + \sqrt5)}{9 - 5} = \dfrac{\cancel{4}^{1}(3 + \sqrt5)}{\cancel{4}\,1} = 3 + \sqrt5$ //

$\dfrac{1}{a} = \dfrac{3 - \sqrt5}{4}$

$a + \dfrac{4}{a} = 3 - \sqrt5 + \dfrac{\cancel{4}^{1}(3 + \sqrt5)}{\cancel{4}\,1} = 3 - \sqrt5 + 3 + \sqrt5 = \underline{6}$

16) $a = \dfrac{3+\sqrt{7}}{2}$

$a^2 = \dfrac{(3+\sqrt{7})^2}{4} = \dfrac{9+7+2\times 3\times \sqrt{7}}{4} = \dfrac{16+6\sqrt{7}}{4} = \dfrac{\cancel{2}^{1}(8+3\sqrt{7})}{\cancel{4}_{2}}$

$\qquad = \dfrac{8+3\sqrt{7}}{2}$

$\dfrac{1}{a} = \dfrac{2}{3+\sqrt{7}} = \dfrac{2(3-\sqrt{7})}{9-(\sqrt{7})^2} = \dfrac{2(3-\sqrt{7})}{9-7} = \dfrac{\cancel{2}(3-\sqrt{7})}{\cancel{2}_{1}} = 3-\sqrt{7}$

$\dfrac{1}{a^2} = (3-\sqrt{7})^2 = 9+7-2\times 3\times \sqrt{7} = 16-6\sqrt{7}$

$\therefore\ a^2 + \dfrac{1}{a^2} = \dfrac{8+3\sqrt{7}}{2} + 16 - 6\sqrt{7}$

$\qquad = \dfrac{8+3\sqrt{7}+32-12\sqrt{7}}{2} = \dfrac{40-9\sqrt{7}}{2}$