IX Homework-2

1) If $x = 4-\sqrt{15}$, then find the value of $\left(x+\frac{1}{x}\right)^2$

2) Prove that $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1$

3) Find the value of a and b if $\frac{\sqrt{2}+1}{\sqrt{2}-1} - \frac{\sqrt{2}-1}{\sqrt{2}+1} = a+\sqrt{2}b$

4) If $x = \sqrt{2}-1$, then find the value of $\left(x-\frac{1}{x}\right)^3$

5) If $\sqrt{2} = 1.414$, then find the value of $\frac{1}{\sqrt{2}+1}$

6) Rationalising factor of $\frac{1}{\sqrt{x}}$ is —

7) Rationalising factor of $\frac{1}{\sqrt{x}+\sqrt{5}}$ is —

8) Rationalise the denominator (i) $\frac{1}{\sqrt{50}}$ (ii) $\frac{2}{3\sqrt{3}}$

9) Show that $\frac{\left(x^{a+b}\right)^2 \cdot \left(x^{b+c}\right)^2 \cdot \left(x^{c+a}\right)^2}{\left(x^a x^b x^c\right)^4} = 1$

10) Evaluate : $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{9}{25}\right)^{\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

11) Find x if $\left(\frac{2}{3}\right)^x \cdot \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$

12) Simplify : $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

13) Simplify : $\left(\frac{1}{2}\right)^{-2} + \left(\frac{2}{3}\right)^{-2} + \left(\frac{3}{4}\right)^{-2}$

14) $\sqrt{10} \times \sqrt{15} =$ (a) $6\sqrt{5}$ (b) $5\sqrt{6}$ (c) $\sqrt{25}$ (d) $10\sqrt{5}$

15) Evaluate : $\frac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}-3^{39}}$

16) Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}}$

17) Show that $\left(x^{a-b}\right)^{a+b} \cdot \left(x^{b-c}\right)^{b+c} \cdot \left(x^{c-a}\right)^{c+a} = 1$

18) Simplify $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

IX Homework-2 (Answers)

1) $x = 4-\sqrt{15}$

$\frac{1}{x} = \frac{1}{(4-\sqrt{15})} \times \frac{(4+\sqrt{15})}{(4+\sqrt{15})} = \frac{4+\sqrt{15}}{4^2-(\sqrt{15})^2}$ $[\because (a+b)(a-b)=a^2-b^2]$

$= \frac{4+\sqrt{15}}{16-15} = 4+\sqrt{15}$

$\therefore \left(x+\frac{1}{x}\right)^2 = \left(4-\cancel{\sqrt{15}}+4+\cancel{\sqrt{15}}\right)^2 = 8^2 = \underline{\underline{64}}$

2) LHS, $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1}$

$= \frac{3-\sqrt{7}}{(3+\sqrt{7})(3-\sqrt{7})} + \frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} + \frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + \frac{\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{3-\sqrt{7}}{3^2-(\sqrt{7})^2} + \frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7})^2-(\sqrt{5})^2} + \frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{\sqrt{3}-1}{(\sqrt{3})^2-1^2}$

$= \frac{3-\sqrt{7}}{9-7} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{3}-1}{3-1}$

$= \frac{3-\sqrt{7}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2}$

$= \frac{3-\cancel{\sqrt{7}}+\cancel{\sqrt{7}}-\cancel{\sqrt{5}}+\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-1}{2} = \frac{3-1}{2} = \frac{2}{2} = \underline{\underline{1}}$, RHS

3) $\frac{\sqrt{2}+1}{\sqrt{2}-1} - \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$\Rightarrow \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)} - \frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{2+1+2\sqrt{2}}{2-1} - \frac{2+1-2\sqrt{2}}{2-1}$

$= \frac{(3+2\sqrt{2})-(3-2\sqrt{2})}{1}$

$= \cancel{3}+2\sqrt{2}\cancel{-3}+2\sqrt{2}$

$= 4\sqrt{2}$

$= 0+4\sqrt{2}$

On Comparing with $a+\sqrt{2}b$, $\boxed{a=0 \quad b=4}$

4) $x = \sqrt{2} - 1$

$$\frac{1}{x} = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2}+1}{2-1} = \frac{\sqrt{2}+1}{1} = \sqrt{2}+1$$

$$\therefore \left(x - \frac{1}{x}\right)^3 = \left[(\sqrt{2}-1) - (\sqrt{2}+1)\right]^3$$

$$= (\cancel{\sqrt{2}} - 1 - \cancel{\sqrt{2}} - 1)^3 = (-2)^3 = -8$$

5) $$\frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2}-1}{2-1} = \frac{\sqrt{2}-1}{1}$$

$$= \sqrt{2} - 1 = 1.414 - 1 = 0.414$$

6) $\sqrt{\pi}$

7) $\sqrt{\pi} - \sqrt{5}$

8) (i) $$\frac{1}{\sqrt{50}} = \frac{\sqrt{50}}{\sqrt{50} \times \sqrt{50}} = \frac{\sqrt{50}}{50} = \frac{\overset{1}{\cancel{5}}\sqrt{2}}{\underset{10}{\cancel{50}}} = \frac{\sqrt{2}}{10}$$

$$\begin{array}{r|l} 5 & 50 \\ \hline 5 & 10 \\ \hline & 2 \end{array}$$

(ii) $$\frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{3\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3}}{3 \times 3} = \frac{2\sqrt{3}}{9}$$

9) LHS, $$\frac{(x^{a+b})^2 \cdot (x^{b+c})^2 \cdot (x^{c+a})^2}{(x^a \cdot x^b \cdot x^c)^4}$$

$$= \frac{x^{2(a+b)} \cdot x^{2(b+c)} \cdot x^{2(c+a)}}{x^{4a} \cdot x^{4b} \cdot x^{4c}}$$

$$= \frac{x^{2a+2b} \cdot x^{2b+2c} \cdot x^{2c+2a}}{x^{4a} \cdot x^{4b} \cdot x^{4c}}$$

$$= \frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}$$

$$= \frac{\overset{1}{\cancel{x^{4a+4b+4c}}}}{\underset{1}{\cancel{x^{4a+4b+4c}}}} = 1, \text{ RHS}$$

10) $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{9}{25}\right)^{\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

$= \left(\frac{3^4}{2^4}\right)^{-\frac{3}{4}} \times \left[\left(\frac{3^2}{5^2}\right)^{\frac{3}{2}} \div \frac{5^{-3}}{2^{-3}}\right]$

$= \frac{3^{\cancel{4}\times -\frac{3}{\cancel{4}}}}{2^{\cancel{4}\times -\frac{3}{\cancel{4}}}} \times \frac{3^{\cancel{2}\times\frac{3}{\cancel{2}}}}{5^{\cancel{2}\times\frac{3}{\cancel{2}}}} \div \frac{2^3}{5^3}$

$= \frac{3^{-3}}{2^{-3}} \times \frac{3^3}{5^3} \times \frac{5^3}{2^3}$

$= \frac{\cancel{2^3}}{\cancel{3^3}} \times \frac{\cancel{3^3}}{\cancel{5^3}} \times \frac{\cancel{5^3}}{\cancel{2^3}} = \underline{\underline{1}}$

11) $\left(\frac{2}{3}\right)^{x} \cdot \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$

$\Rightarrow \left(\frac{3}{2}\right)^{-x} \times \left(\frac{3}{2}\right)^{2x} = \frac{3^4}{2^4}$

$\Rightarrow \left(\frac{3}{2}\right)^{-x+2x} = \left(\frac{3}{2}\right)^{4}$

$\therefore -x+2x = 4$

$\boxed{x = 4}$

12) $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

$= \left[5\left(2^{\cancel{3}\times\frac{1}{\cancel{3}}} + 3^{\cancel{3}\times\frac{1}{\cancel{3}}}\right)^3\right]^{\frac{1}{4}}$

$= \left[5\,(2+3)^3\right]^{\frac{1}{4}} = \left(5\times5^3\right)^{\frac{1}{4}} = \left(5^4\right)^{\frac{1}{4}} = 5^{\cancel{4}\times\frac{1}{\cancel{4}}} = \underline{\underline{5}}$

13) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{2}{3}\right)^{-2} + \left(\frac{3}{4}\right)^{-2}$

$= 2^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{4}{3}\right)^2 = 4^{\times 36} + \frac{9^{\times 9}}{4\times 9} + \frac{16^{\times 4}}{9\times 4}$

$= \frac{144+81+64}{36} = \underline{\underline{\frac{289}{36}}}$

14) $\sqrt{10} \times \sqrt{15} = \sqrt{150} = 5\sqrt{6}$ (b)

$$\begin{array}{r|l} 3 & 150 \\ 5 & 50 \\ 5 & 10 \\ & 2 \end{array}$$

15) $$\frac{3^{40} + 3^{39} + 3^{38}}{3^{41} + 3^{40} - 3^{39}}$$

$$= \frac{\cancel{3^{38}}\,(3^2 + 3^1 + 1)}{\cancel{3^{38}}\,(3^3 + 3^2 - 3^1)} = \frac{9+3+1}{27+9-3} = \underline{\underline{\frac{13}{33}}}$$

16) $$\frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}}$$

$$= \frac{4}{6^{\cancel{3} \times -\frac{2}{\cancel{3}}}} - \frac{1}{4^{\cancel{4} \times -\frac{3}{\cancel{4}}}} = \frac{4}{6^{-2}} - \frac{1}{4^{-3}}$$

$= 4 \times 6^2 - 4^3$

$= 4 \times 36 - 64$

$= 144 - 64 = \underline{\underline{80}}$

17) LHS, $\left(x^{a-b}\right)^{a+b} \cdot \left(x^{b-c}\right)^{b+c} \cdot \left(x^{c-a}\right)^{c+a}$

$= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)}$

$= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2}$

$= x^{\cancel{a^2} - \cancel{b^2} + \cancel{b^2} - \cancel{c^2} + \cancel{c^2} - \cancel{a^2}} = x^0 = \underline{\underline{1}}$, RHS

18) $\sqrt[4]{81} - 8 \times \sqrt[3]{216} + 15 \times \sqrt[5]{32} + \sqrt{225}$

$= 3^{\cancel{4} \times \frac{1}{\cancel{4}}} - 8 \times 6^{\cancel{3} \times \frac{1}{\cancel{3}}} + 15 \times 2^{\cancel{5} \times \frac{1}{\cancel{5}}} + 15$

$= 3 - 8 \times 6 + 15 \times 2 + 15$

$= 3 - 48 + 30 + 15$

$= 48 - 48$

$= \underline{\underline{0}}$